Aspects of nucleon Compton scattering on the lattice

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AA, Ulf-G. Meißner and A. Rusetsky, Phys. Rev. D **95**, 031502 (2017)

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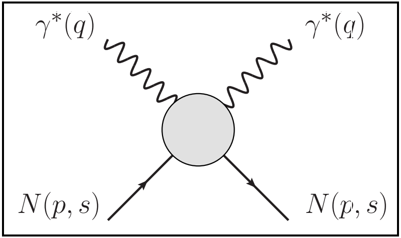
Outline

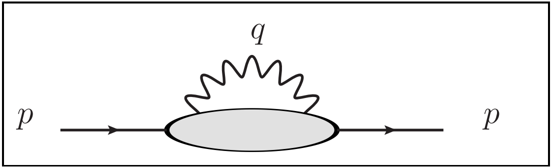
- Introduction: Compton scattering
- Nucleon in an external field
- Energy shift
- Discussion of the result
- Outlook

Compton scattering



- Two invariant amplitudes: $T_1(\nu, q^2)$ and $T_2(\nu, q^2)$, $\quad \nu \equiv p \cdot q/m$
- Why important?
    - ▷ the proton-neutron mass difference
    - ▷ Lamb shift in the muonic hydrogen
    - ▷ the existence of a fixed pole in the Regge theory

Cottingham formula



- Electromagnetic contribution to the proton-neutron mass difference:

$$(m_p - m_n)_{\mathrm{em}} = \frac{ie^2}{2m(2\pi)^4} \int^{\Lambda} d^4q D(q^2)\{3q^2\, T_1 + (2\nu^2 + q^2)\, T_2\} + \text{counter terms}$$

  ▷ $D(q^2)$ - photon propagator

  Cottingham (63)

- Electroproduction cross sections $\rightarrow$ T_1, T_2 $\quad (Q^2 = -q^2 > 0)$

Information on $T_1(\nu, Q^2)$

- $T_1(\nu, Q^2) \to$ once-subtracted dispersion relation (fixed Q^2)
- A problem: $S_1(Q^2) \equiv T_1(0, Q^2)$ is not fixed by experiment

$$S_1(Q^2) = S_1^{el}(Q^2) + S_1^{inel}(Q^2)$$

▷ $S_1^{el}(Q^2) \to$ Born terms (+ nucleon FFs)

▷ Low-energy theorem: $S_1^{inel}(0) = -\frac{\kappa^2}{4m^2} - \frac{m}{\alpha_{\rm em}}\,\beta_M$ Scherer, Tarrach

$\hookrightarrow \beta_M$ – magnetic polarizability, $\kappa = F_2(0)$, $\alpha_{\rm em} \approx 1/137$

▷ OPE: $S_1(Q^2 \to \infty) = \frac{C}{Q^4}$, C – known coefficient Collins, Hill

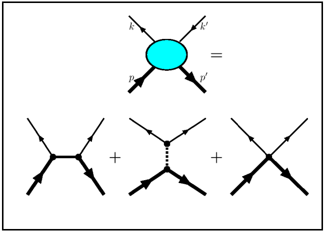
- Intermediate region $0 < Q^2 \lesssim 2$ GeV^2: $S_1^{inel}(Q^2)$ is unknown

Reggeon dominance hypothesis

- No fixed pole $\rightarrow$ energy-independent contribution (Reggeon exchange):

$$T_1(\nu, Q^2) - T_1^R(\nu, Q^2) \rightarrow 0, \quad \nu \rightarrow \infty \, \text{at fixed } Q^2$$

Gasser and Leutwyler (75)



Source: S. Brodsky et al., Phys.Rev. D79, 033012 (2009)

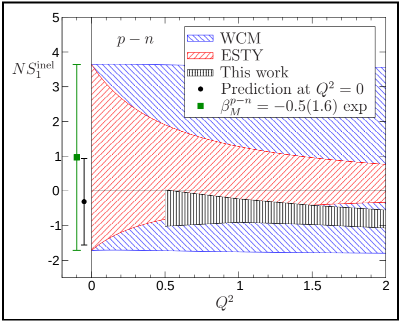
▷ The s-channel amplitude

$$A_s(s, M) = \frac{M^2}{M^2 - s}$$

$$A_s(s < M^2, M) = \sum_{J \geq 0} (-1)^J \left(-\frac{s}{M^2}\right)^J$$

▷ $J = 0 \rightarrow$ a fixed pole

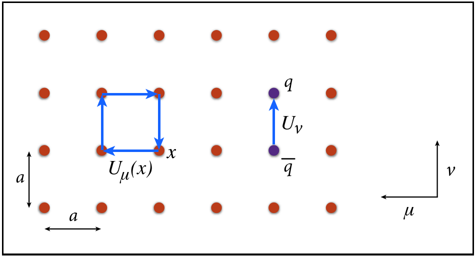
Theoretical approaches



Source: J. Gasser et al., Eur. Phys. J. C **75**, 375 (2015)

- ▷ Chiral EFTs, NRQED — BKM, Hill, Pineda ...
- ▷ Phenomenological Ansatzes — Pachucki, Walker-Loud, ...
- ▷ Reggeon dominance hypothesis — Gasser, Leutwyler, ...
- Lattice QCD $\rightarrow$ devoid of any model dependence

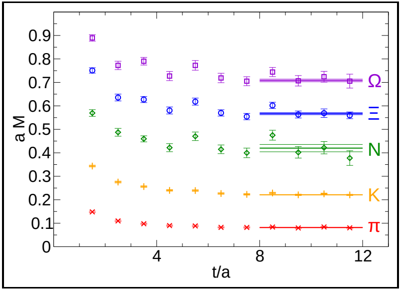
Lattice QCD



- Path integral formulation in Euclidean space-time
- Space-time is discretized and finite ⇒ natural UV cut-off $\sim 1/a$
  K. G. Wilson (74)
- Numerical integration → Monte Carlo methods
- Correlation functions → energy levels, form factors

↪ review by FLAG: S. Aoki *et al.*, arXiv:1607.00299 (2016)

Stable hadrons



Source: BMW Collaboration, Science **322**, 1224 (2008)

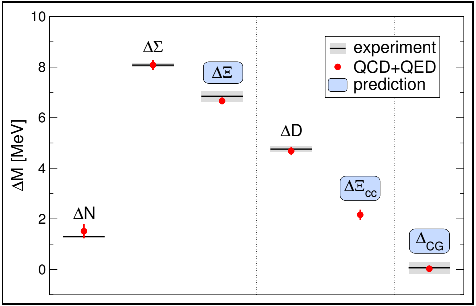
- Two-point correlation function:

$$C(t) = \sum_{\mathbf{x}} \langle 0 | O(\mathbf{x}, t) O^{\dagger}(\mathbf{0}, 0) | 0 \rangle = |Z_O|^2 e^{-mt} \left[1 + \sum_n |Z_n|^2 e^{-\Delta E_n t} \right]$$

  ▷ $O(\mathbf{x}, t)$ - field operator, Z_O, Z_n - overlap factors, $\Delta E_n > 0$

- Effective mass: $M(t) = \frac{1}{a} \log \frac{C(t)}{C(t+a)}$, $\quad M(t) \to m$

Lattice QCD+QED



Source: S. Borsanyi et al., Science **347** (2015) 1452

	Δm [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)

- The fully unquenched lattice QCD + QED computation
  - ▷ 4 non-degenerate flavors, $m_\pi \approx 195$ MeV

BMW Collaboration

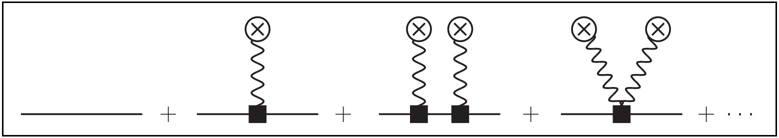
External field method

How to study the Compton scattering on the lattice?

- four-point function $\to$ Compton tensor

$$T^{\mu\nu}(p,q) = \frac{1}{2}\sum_s \frac{i}{2}\int d^4x e^{iq\cdot x}\langle p,s|Tj^\mu(x)j^\nu(0)|p,s\rangle$$

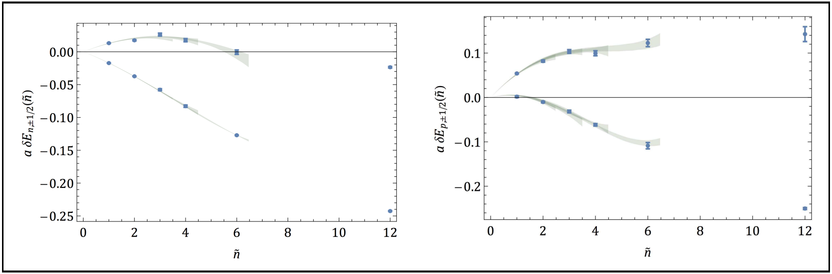
- two-point function in an external em. field $A_\mu(x)$



Source: W. Detmold et al., Phys. Rev. D **73**, 114505 (2006)

- Uniform magnetic field $\to$ polarizabilities — NPLQCD
- Static magnetic field $\to$ energy levels, measured on the lattice

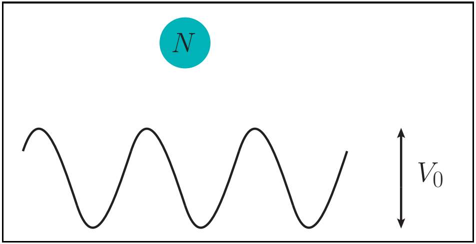
Magnetic properties



Source: E. Chang *et al.* [NPLQCD Collaboration], Phys. Rev. D **92**, 114502 (2015)

- Lattice calculation at one SU(3)-symm. point, $m_\pi \approx 806$ MeV
- Constant magnetic field: $\mathbf{B} = \frac{6\pi}{eL^2}\tilde{n}\,\mathbf{e}_z\,, \quad \tilde{n} \in \mathbb{Z}$ 't Hooft (79)

$\gamma^* N \to \gamma^* N$: field configuration



- Static periodic magnetic field $\mathbf{B} = (0, 0, B_3)$:

$$A_1 = \frac{B}{\omega}\sin(\omega x_2), \qquad A_0 = A_2 = A_3 = 0$$

- Frequency $\omega \neq 0 \to$ photon virtuality $q^2 = -\omega^2$
- Magnetic flux is quantized $\Rightarrow \omega = \frac{2\pi N}{L}, \qquad N \in \mathbb{Z}\backslash\{0\}$

Energy shift

- For $\nu = 0$ and $q^2 < 0 \to S_1(q^2)$ is real
- If $V_0 = e^2 B^2 / 2m\omega^2$ is "small" $\Rightarrow$ perturbation theory
- The free energy spectrum:

$$w(\mathbf{k}_n) = \sqrt{m^2 + \mathbf{k}_n^2}, \qquad \mathbf{k}_n = \frac{2\pi\mathbf{n}}{L}, \qquad \mathbf{n} \in \mathbb{Z}^3$$

- Spin-averaged energy shift of the ground state ($\mathbf{k}_n = 0$):

$$\delta E = -\frac{B^2}{4m\omega^2}\, T^{11}(0,\omega) + O(B^3) = \frac{B^2}{4m}\, S_1(-\omega^2) + O(B^3)$$

$\hookrightarrow$finite-volume effects are neglected

Zero frequency limit $\omega \to 0$

- Energy spectrum $\to$ Landau levels – non-perturbative transition
- $S_1^{el}(-\omega^2)$ explodes: $S_1^{el}(-\omega^2) \sim 1/\omega^2$ (propagator)
- Quantization of the magnetic flux:
  - $\omega = \frac{2\pi N}{L}$ – quantized: $L \to \infty$ – inconvenient
  - $B = \frac{6\pi N}{eL^2} \frac{\omega L}{\sin(\omega L)}, \quad \omega$ – arbitrary
- Lower bound: $\omega^2_{min} = 0.13\ \mathrm{GeV}^2$ (p), $\omega^2_{min} = 0.06\ \mathrm{GeV}^2$ (n)
- Numerical analysis is required

Outlook

- Energy shift in a finite volume ✓
- Numerical analysis of the constant field limit
- Disconnected contributions in partially quenched χPT
- Inclusive semi-leptonic B meson decays

$$T_{\mu\nu} \propto i \int d^4 x e^{iqx} \langle B | T J_\mu^\dagger(x) J_\nu(0) | B \rangle \to \text{structure functions}$$

$\hookrightarrow$ J_μ – electroweak current for $b \to c l \nu$