

Dispersive analysis of light-meson transition form factors

Bastian Kubis

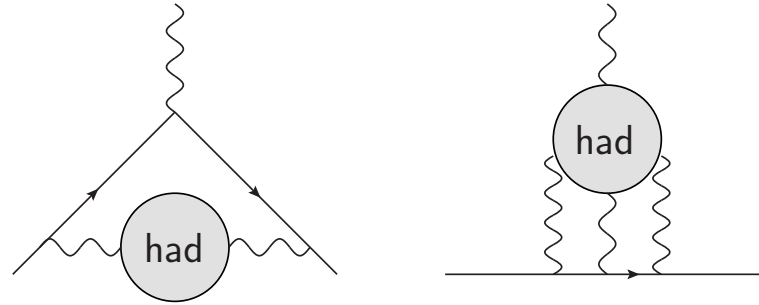
HISKP (Theorie) & BCTP
Universität Bonn, Germany

Physics of the Standard Model and Beyond

Tbilisi, 25/9/2017

Hadronic light-by-light scattering

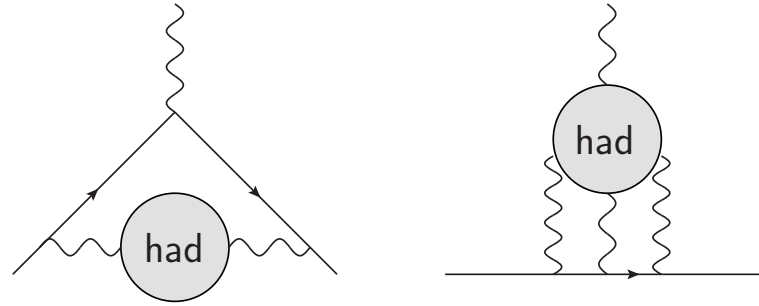
- leading and next-to-leading hadronic effects in $(g - 2)_\mu$:



- hadronic vacuum polarisation: $e^+e^- \rightarrow \text{hadrons}$
- hadronic light-by-light soon dominant uncertainty

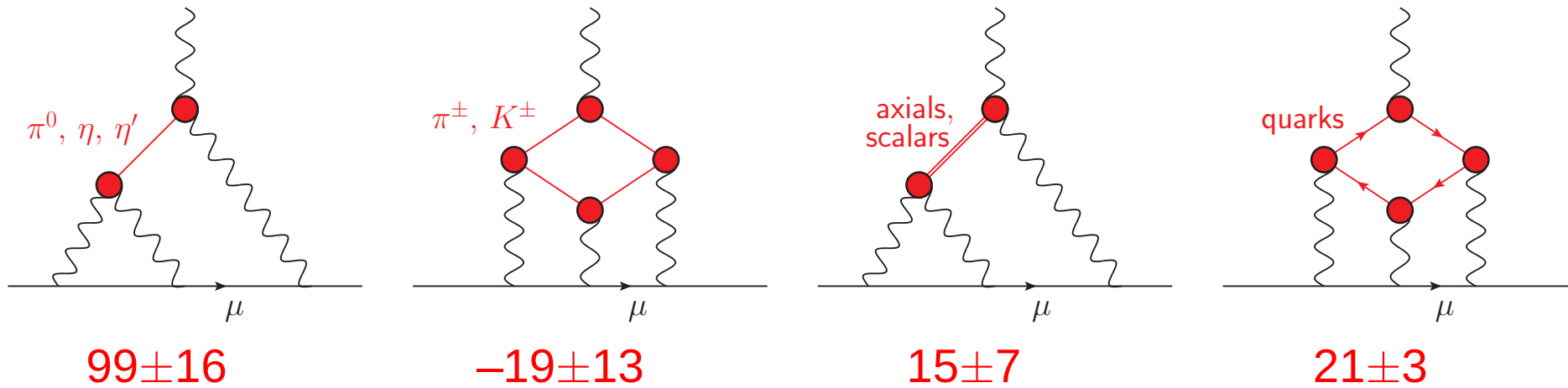
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- different contributions estimated (in 10^{-11}):



→ how to control hadronic **modelling**?

Jegerlehner, Nyffeler 2009

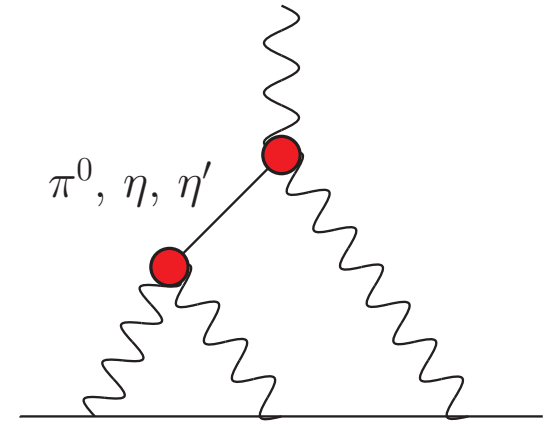
Pseudoscalar transition form factors and $(g - 2)_\mu$

- largest individual HLbL contribution:

pseudoscalar pole terms

singly / doubly virtual form factors

$F_{P\gamma\gamma^*}(q^2, 0)$ and $F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$



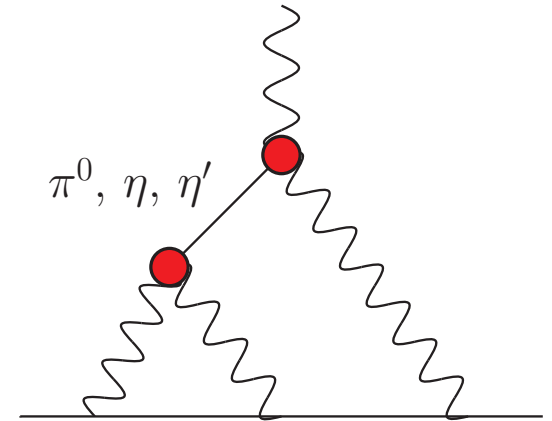
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- normalisation fixed by Wess–Zumino–Witten anomaly, e.g.:

$$F_{\pi^0\gamma\gamma}(0, 0) = \frac{e^2}{4\pi^2 F_\pi}$$

F_π : pion decay constant $\longrightarrow$ measured at 1.5% level PrimEx 2011

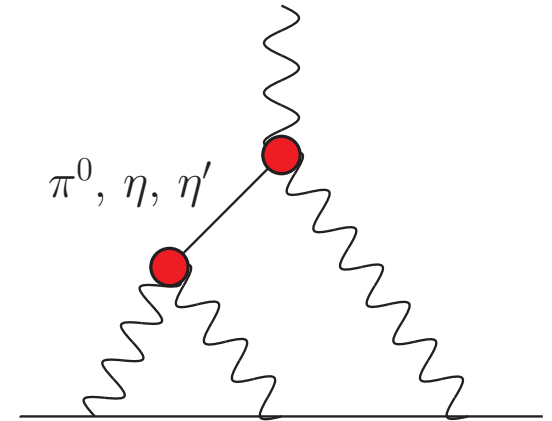
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- q_i^2 -dependence: often modelled by vector-meson dominance
 - $\longrightarrow$ what can we learn from analyticity and unitarity constraints?
 - $\longrightarrow$ what experimental input sharpens these constraints?

Dispersive analysis of $\pi^0/\eta \rightarrow \gamma^* \gamma^*$

- isospin decomposition:

$$F_{\pi^0 \gamma^* \gamma^*}(q_1^2, q_2^2) = F_{vs}(\textcolor{red}{q}_1^2, \textcolor{blue}{q}_2^2) + F_{vs}(\textcolor{red}{q}_2^2, \textcolor{blue}{q}_1^2)$$

$$F_{\eta \gamma^* \gamma^*}(q_1^2, q_2^2) = F_{vv}(\textcolor{red}{q}_1^2, \textcolor{red}{q}_2^2) + F_{ss}(\textcolor{blue}{q}_1^2, \textcolor{blue}{q}_2^2)$$

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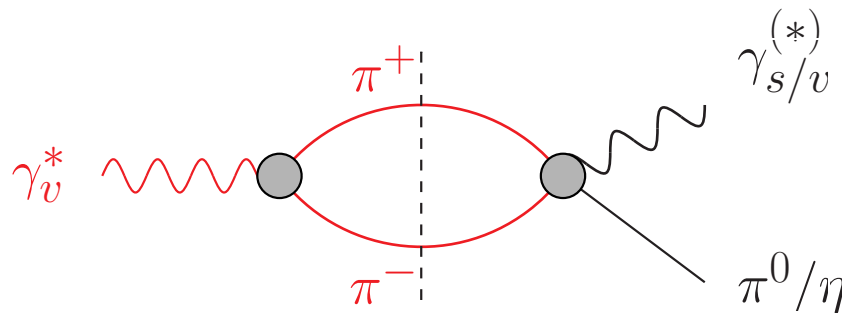
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- analyse the leading hadronic intermediate states:

Hanhart et al. 2013, Hoferichter et al. 2014



- ▷ **isovector** photon: **2 pions**

$\propto$ pion vector form factor $\times \gamma\pi \rightarrow \pi\pi / \eta \rightarrow \pi\pi\gamma$

all determined in terms of pion–pion P-wave phase shift

Dispersive analysis of $\pi^0/\eta \rightarrow \gamma^* \gamma^*$

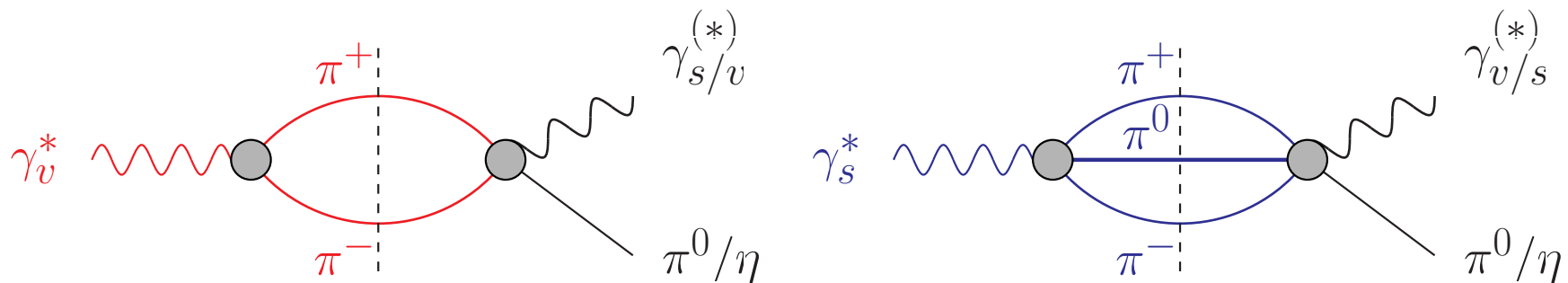
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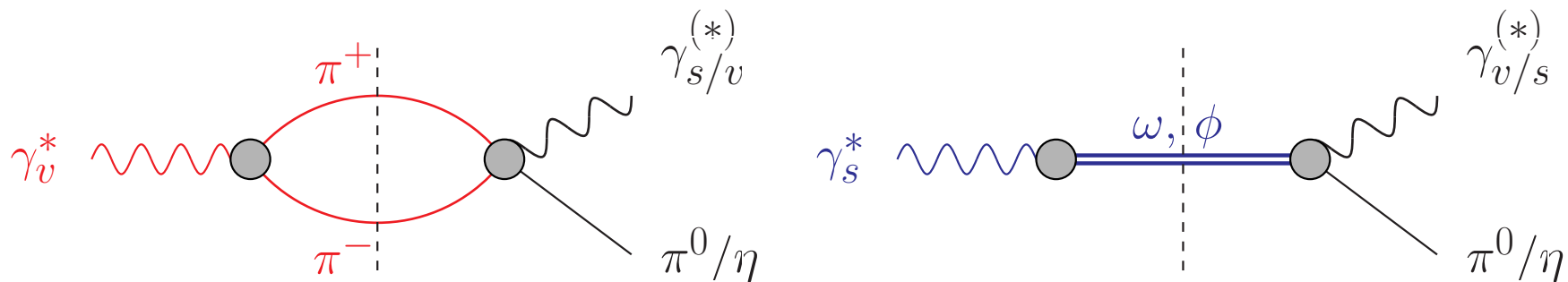
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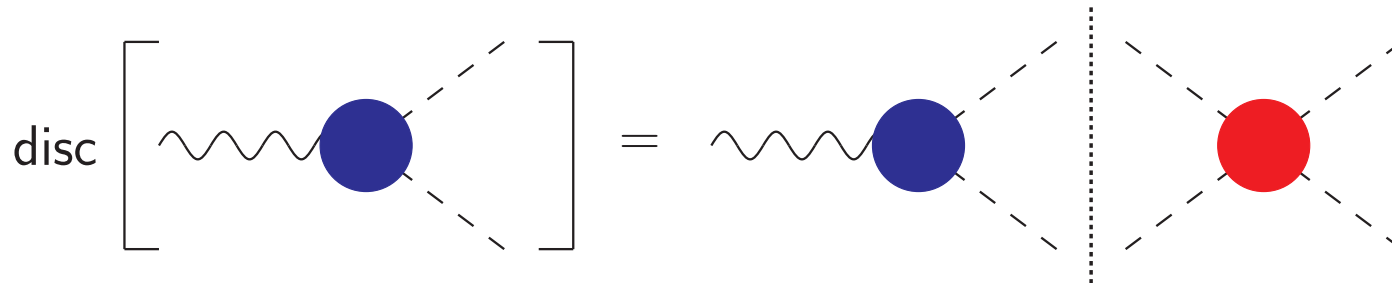
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- ▷ **isoscalar** photon: **3 pions** $\rightarrow$ dominated by narrow ω, ϕ

$\leftrightarrow \omega/\phi$ transition form factors; very small for the η

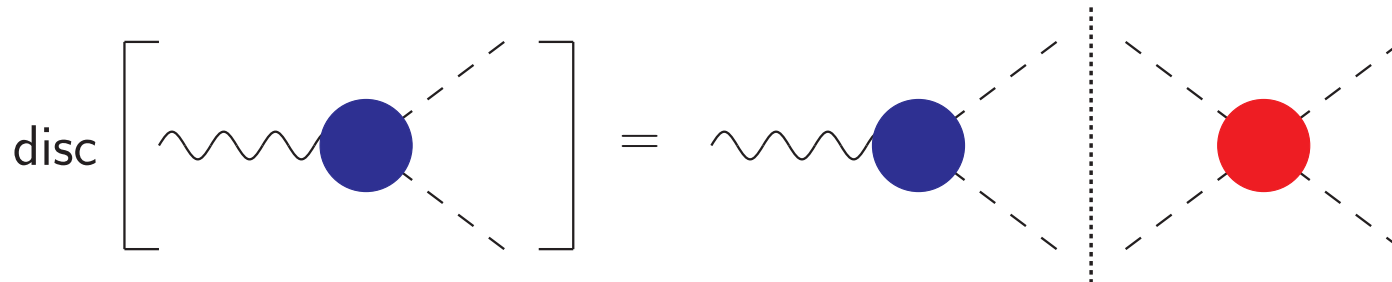
Warm-up: charged pion form factor



$$\frac{1}{2i} \text{disc } F_{\pi}^V(s) = \text{Im } F_{\pi}^V(s) = F_{\pi}^V(s) \times \theta(s - 4 M_{\pi}^2) \times \sin \delta_1^1(s) e^{-i\delta_1^1(s)}$$

→ **final-state theorem**: phase of $F_{\pi}^V(s)$ is just $\delta_1^1(s)$ **Watson 1954**

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→ **final-state theorem**: phase of $F_{\pi}^V(s)$ is just $\delta_1^1(s)$ **Watson 1954**

- solution:

$$F_{\pi}^V(s) = P(s)\Omega(s), \quad \Omega(s) = \exp \left\{ \frac{s}{\pi} \int_{4M_{\pi}^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s' - s)} \right\}$$

$P(s)$ polynomial, $\Omega(s)$ **Omnès function** **Omnès 1958**

▷ $\pi\pi$ phase shifts from Roy equations

Ananthanarayan et al. 2001, García-Martín et al. 2011

▷ $P(0) = 1$ from symmetries (gauge invariance)

- below 1 GeV: $F_{\pi}^V(s) \approx (1 + 0.1 \text{ GeV}^{-2}s)\Omega(s)$

slope due to inelastic resonances ρ' , $\rho'' \dots$

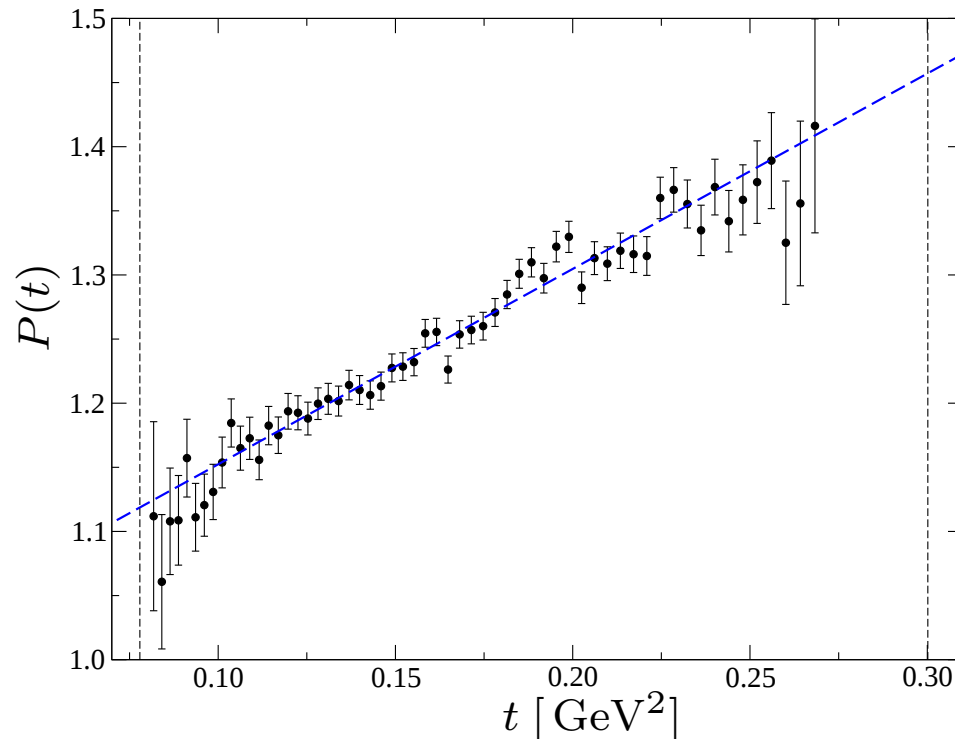
Hanhart 2012

Final-state universality: $\eta, \eta' \rightarrow \pi^+ \pi^- \gamma$

- $\eta^{(\prime)} \rightarrow \pi^+ \pi^- \gamma$ driven by the **chiral anomaly**, $\pi^+ \pi^-$ in P-wave
→ final-state interactions **the same** as for vector form factor
- ansatz: $\mathcal{F}_{\pi\pi\gamma}^{\eta^{(\prime)}} = A \times P(t) \times \Omega(t)$, $P(t) = 1 + \alpha^{(\prime)} t$, $t = M_{\pi\pi}^2$

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- divide data by pion form factor $\rightarrow P(t)$ Stollenwerk et al. 2012

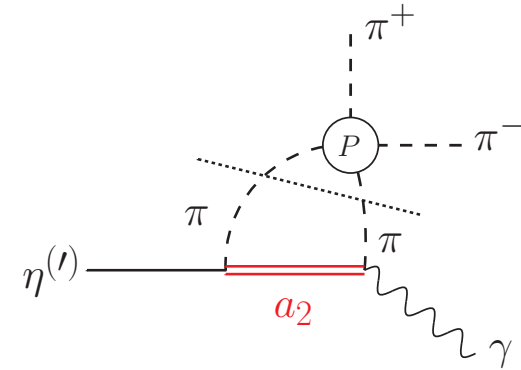
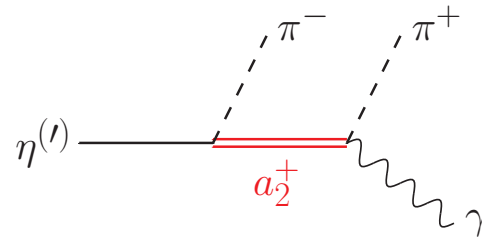
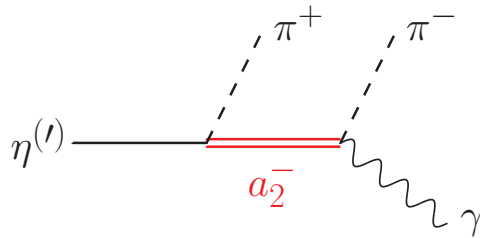


$\rightarrow$ exp.: $\alpha_{\text{KLOE}} = (1.52 \pm 0.06) \text{ GeV}^{-2}$

cf. KLOE 2013

$\eta, \eta' \rightarrow \pi^+ \pi^- \gamma$ with left-hand cuts

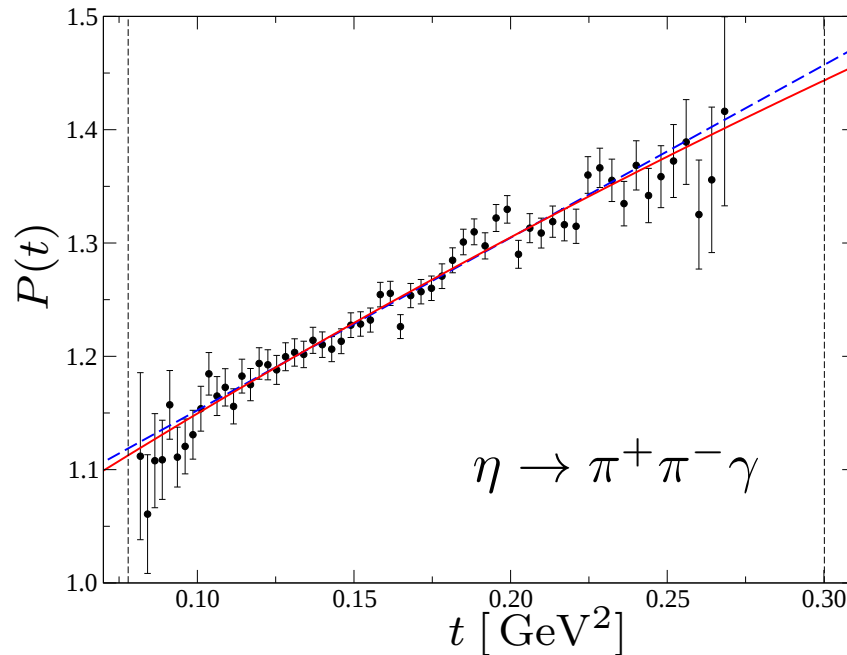
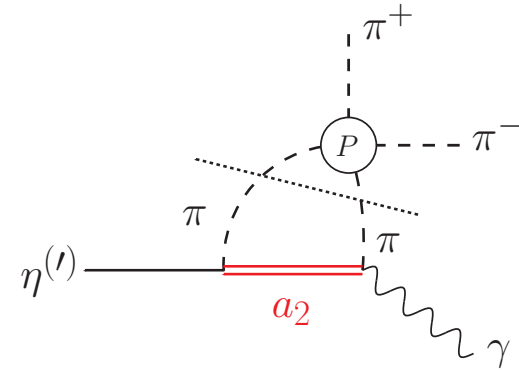
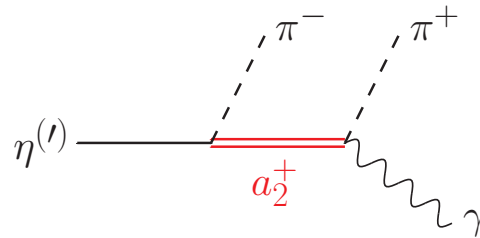
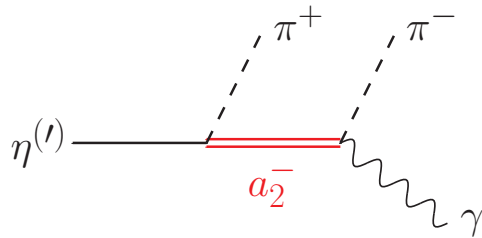
- include a_2 : leading resonance in $\pi\eta^{(\prime)}$



BK, Plenter 2015

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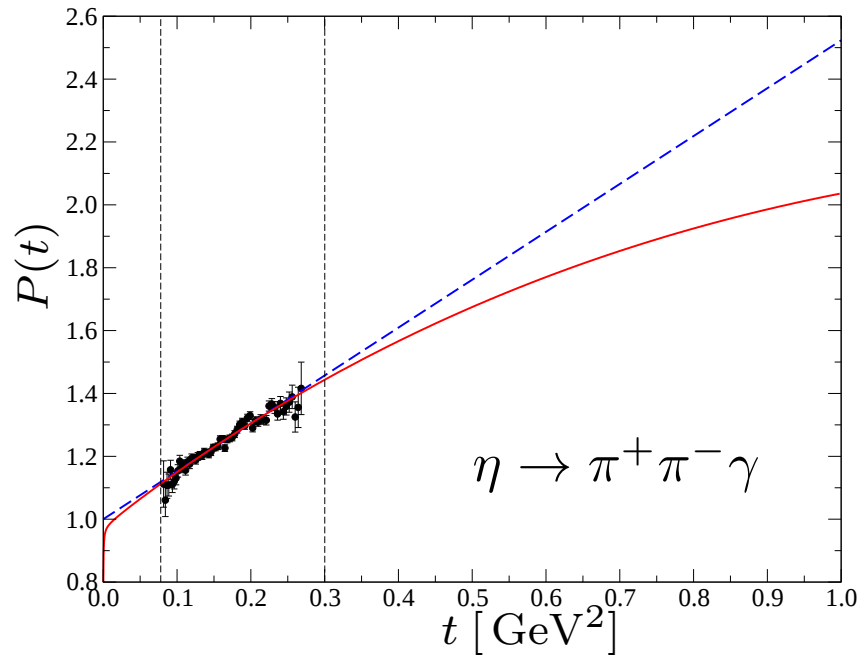
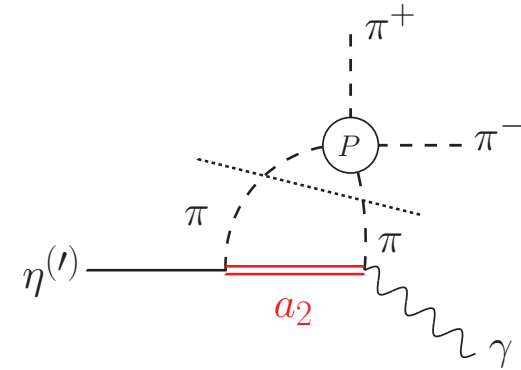
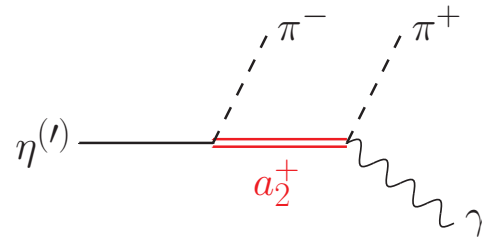
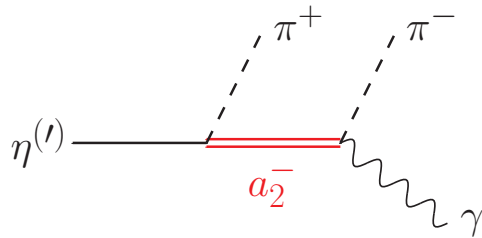


KLOE 2013; BK, Plenter 2015

- induces **curvature** in $P(t)$

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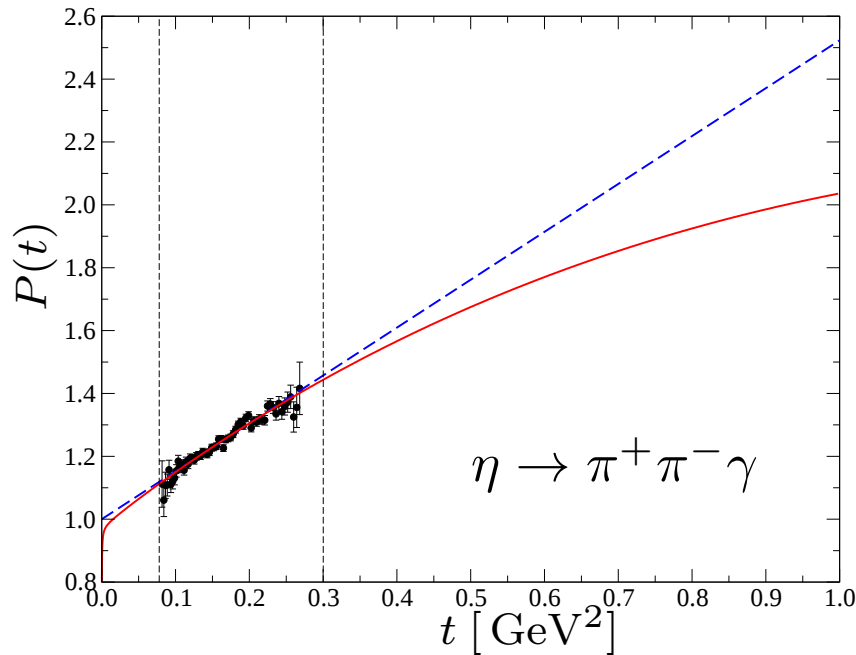
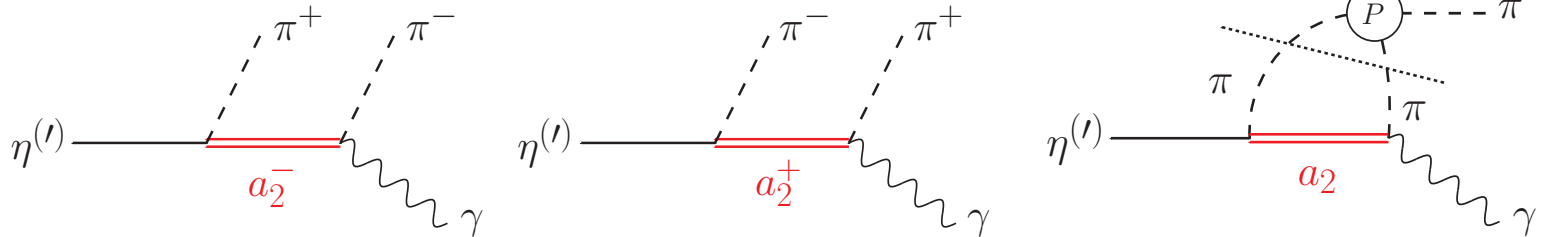


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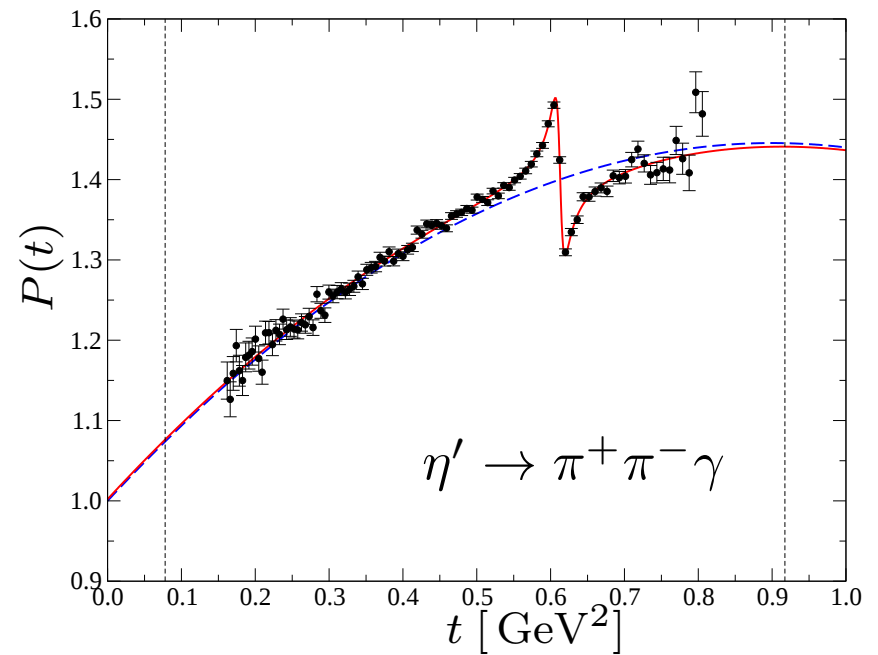
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KLOE 2013; BK, Plenter 2015



BESIII prel.; Hanhart et al. 2017

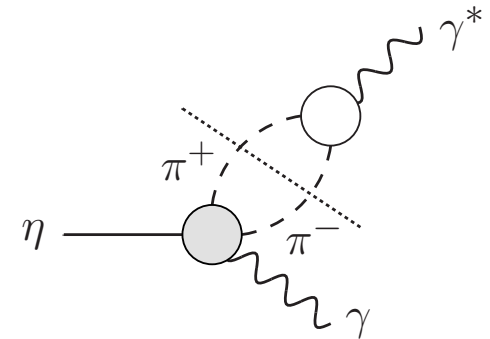
- induces **curvature** in $P(t)$

- curvature**, plus ρ - ω **mixing**

Transition form factor $\eta \rightarrow \gamma^* \gamma$

Hanhart et al. 2013

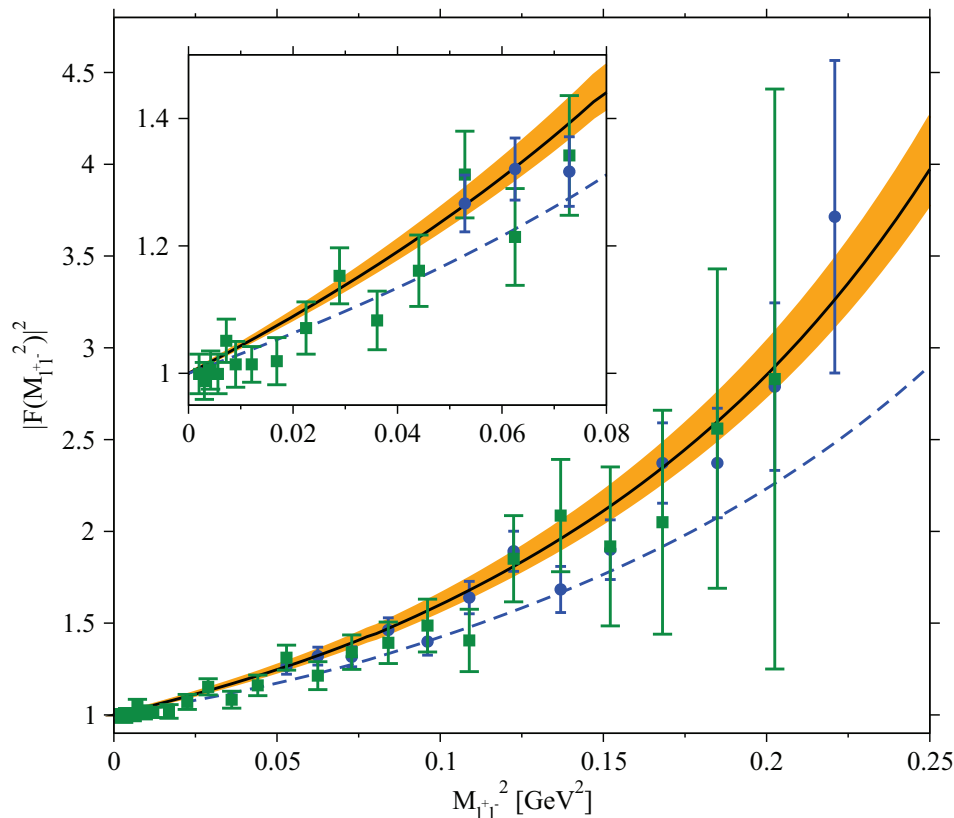
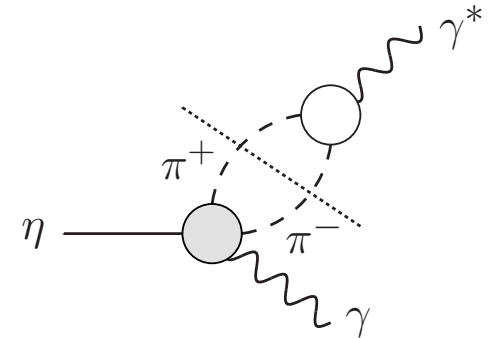
$$\bar{F}_{\eta\gamma^*\gamma}(q^2, 0) = 1 + \frac{\kappa_\eta q^2}{96\pi^2 F_\pi^2} \int_{4M_\pi^2}^{\infty} ds \sigma(s)^3 P_V(s) P_\eta(s) \frac{|\Omega(s)|^2}{s - q^2} \\ + \Delta F_{\eta\gamma^*\gamma}^{I=0}(q^2, 0) [\longrightarrow \text{VMD}]$$



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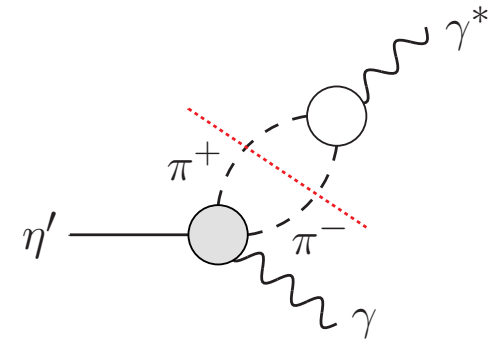


→ huge statistical advantage of using **hadronic input** for $\eta \rightarrow \pi^+ \pi^- \gamma$ over direct measurement of $\eta \rightarrow e^+ e^- \gamma$ (rate suppressed by α_{QED}^2)

figure courtesy of C. Hanhart
data: NA60 2011, A2 2014

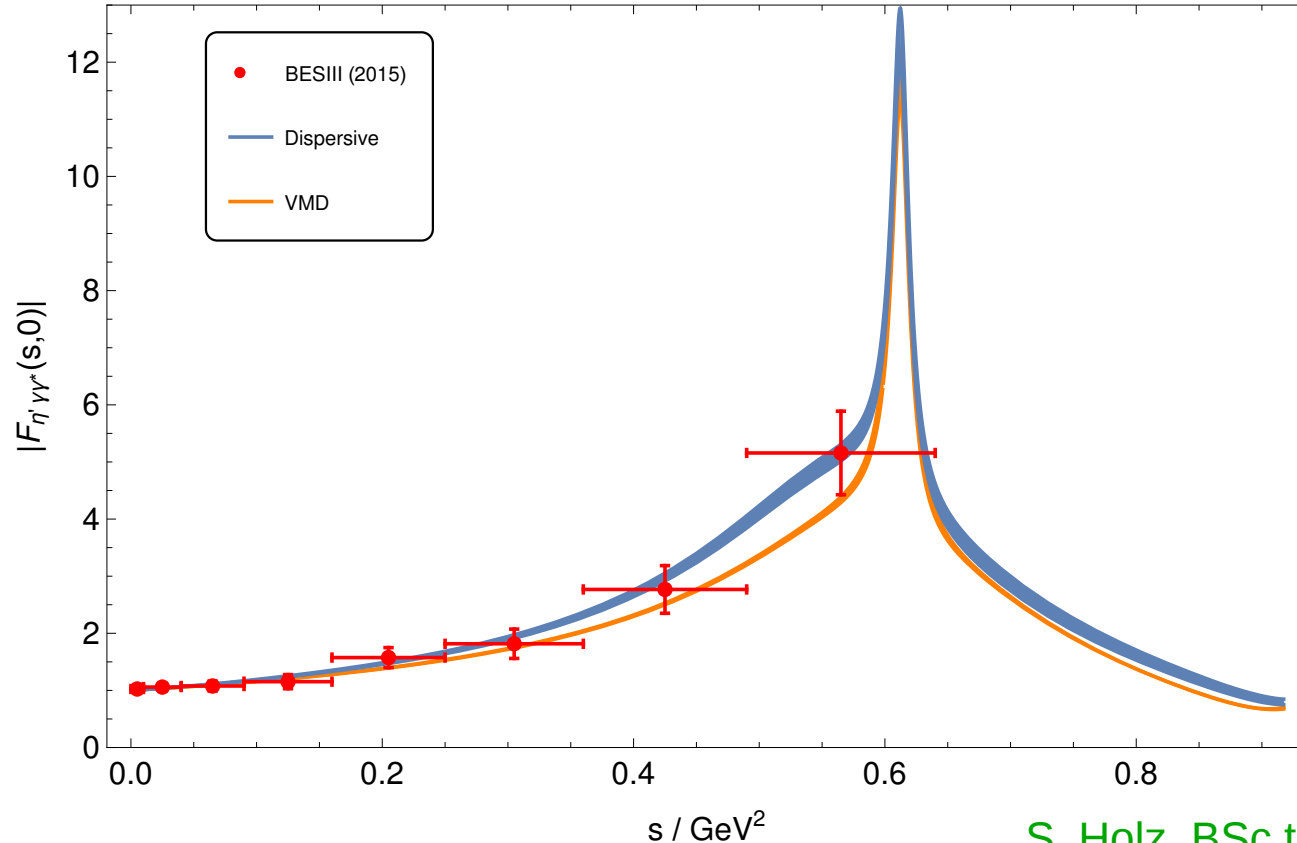
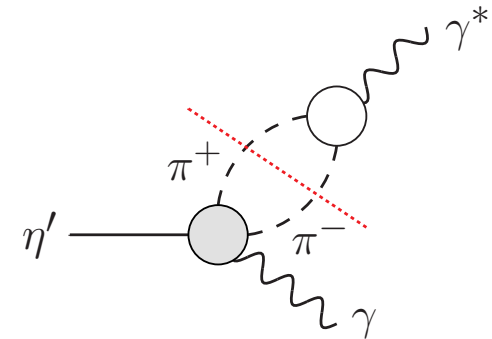
Prediction for η' transition form factor

- **isovector**: combine high-precision data on $\eta' \rightarrow \pi^+ \pi^- \gamma$ and $e^+ e^- \rightarrow \pi^+ \pi^-$
- **isoscalar**: VMD, couplings fixed from $\eta' \rightarrow \omega \gamma$ and $\phi \rightarrow \eta' \gamma$



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S. Holz, BSc thesis 2016

Anomalous process $\gamma\pi \rightarrow \pi\pi$

- $\gamma\pi \rightarrow \pi\pi$: crossing symmetry, WZW low-energy theorem

$$\mathcal{F}(s, t, u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u), \quad \mathcal{F}(0, 0, 0) = F_{3\pi} = \frac{e}{4\pi^2 F_\pi^3}$$

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- left-hand cut $\hat{\mathcal{F}}(s)$ and right-hand cut $\mathcal{F}(s)$ self-consistent:

$$\mathcal{F}(s) = \Omega(s) \left\{ \frac{C_1}{3} + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin \delta_1^1(s') \hat{\mathcal{F}}(s')}{|\Omega(s')|(s' - s)} \right\}$$

$$= \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots$$

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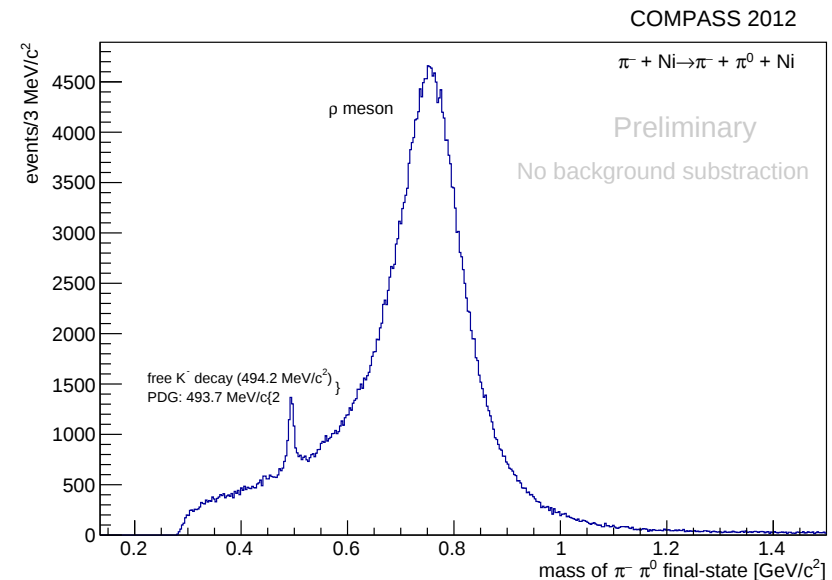
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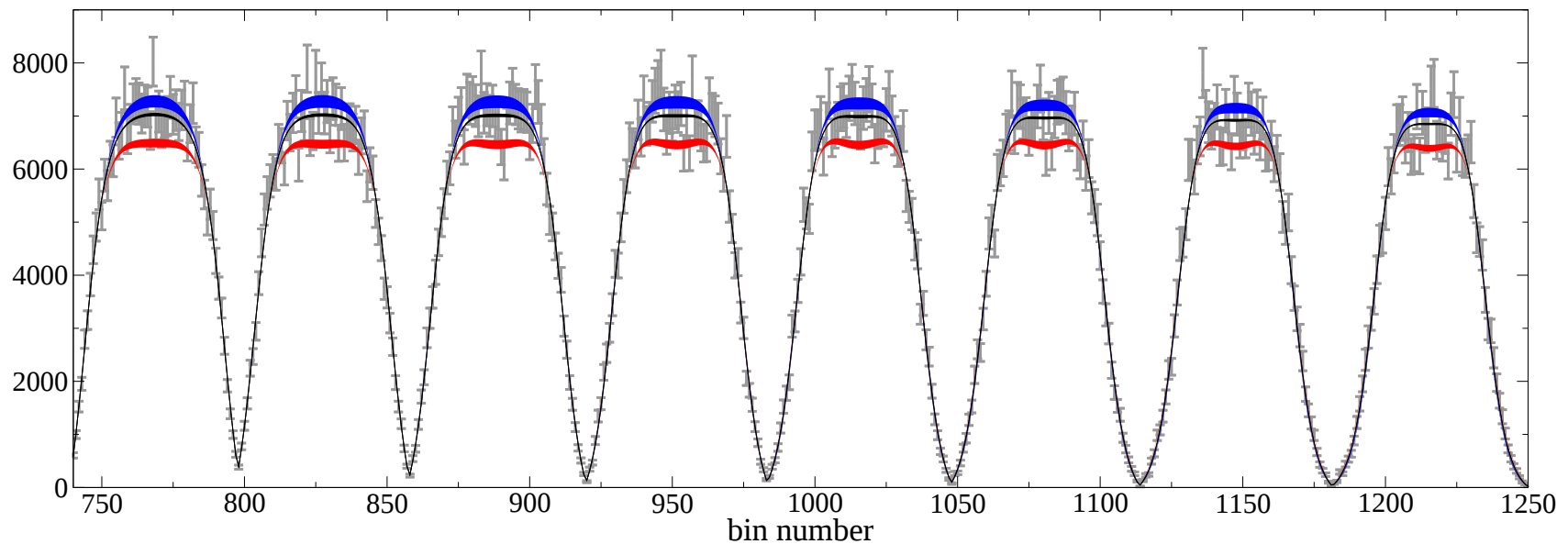
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- high-accuracy data from Primakoff spectrum COMPASS
Seyfried, MSc thesis 2017
- fit dispersive representation to data, extract $F_{3\pi} \simeq C_1$
Hoferichter, BK, Sakkas 2012
- lattice HadSpec coll. 2015



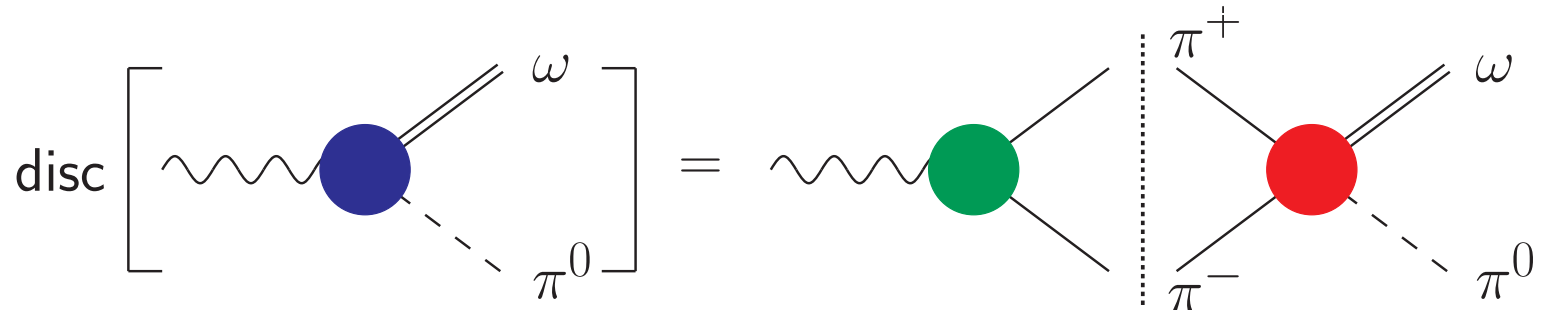
Extension to decays: $\omega/\phi \rightarrow 3\pi$

- same quantum numbers as $\gamma\pi \rightarrow \pi\pi$ Niecknig, BK, Schneider 2012
- first $\omega \rightarrow 3\pi$ Dalitz plot measurement WASA-at-COSY 2017
- test accuracy on $\phi \rightarrow 3\pi$ Dalitz plot: $2 \cdot 10^6$ events KLOE 2005



	$\hat{\mathcal{F}} = 0$	once-subtracted	twice-subtracted
χ^2/ndof	1.71 ... 2.06	1.17 ... 1.50	1.02 ... 1.03

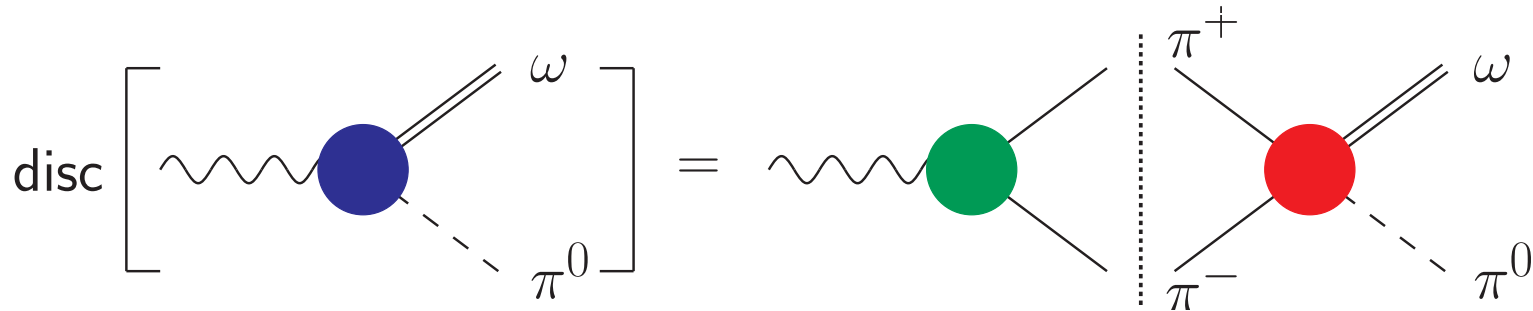
Transition form factor $\omega(\phi) \rightarrow \pi^0 \ell^+ \ell^-$



- ω transition form factor related to

pion vector form factor $\times$ $\omega \rightarrow 3\pi$ decay amplitude

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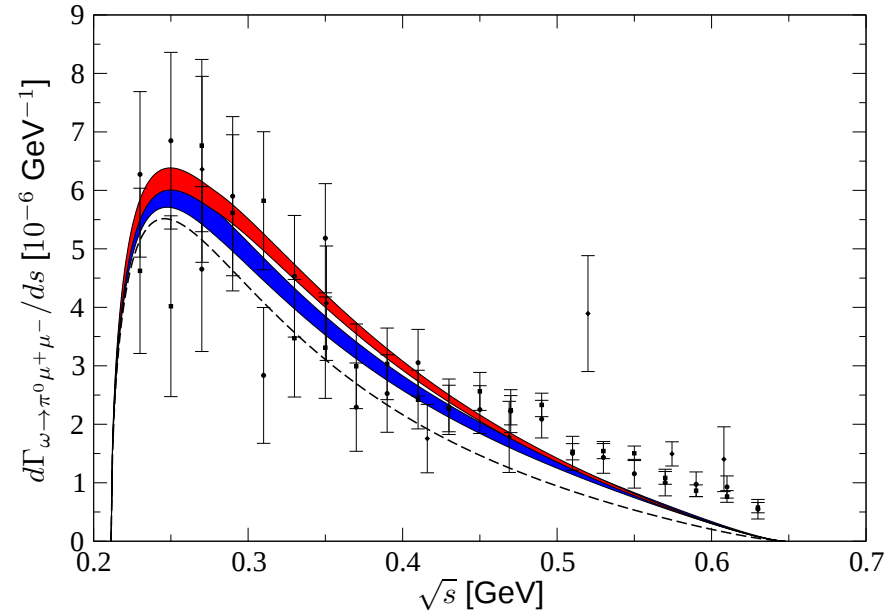
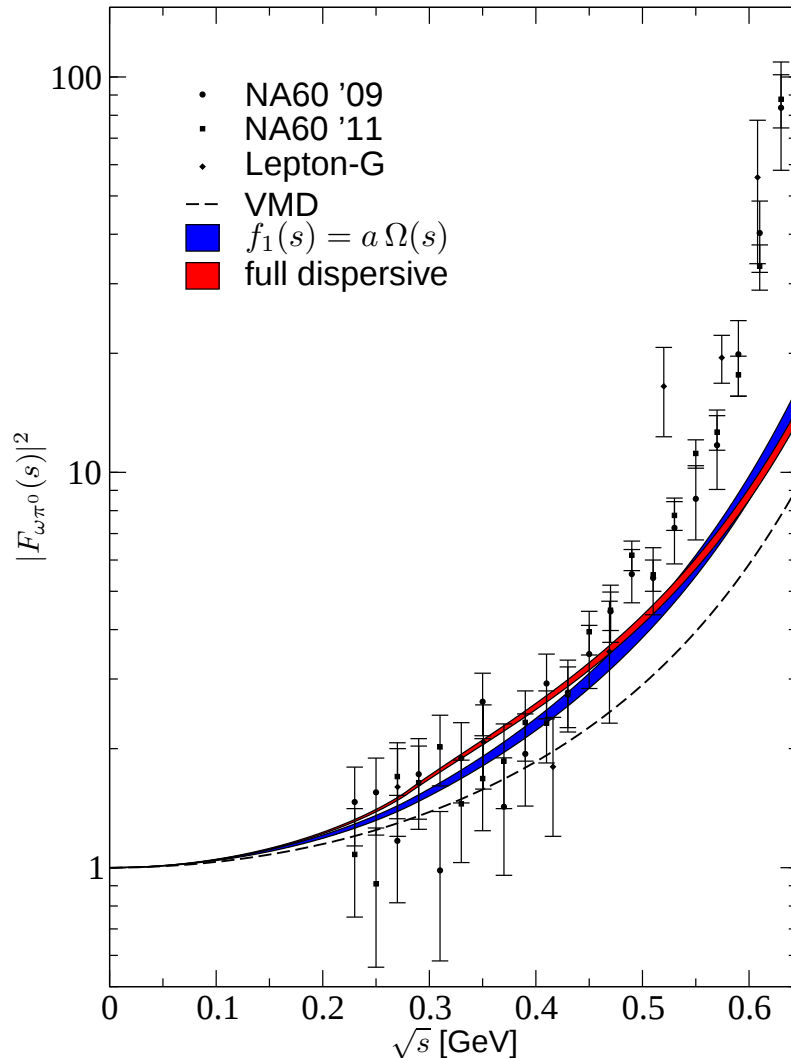
pion vector form factor $\times$ $\omega \rightarrow 3\pi$ decay amplitude

- form factor normalization yields rate $\Gamma(\omega \rightarrow \pi^0 \gamma)$
(2nd most important ω decay channel)

→ works at 95% accuracy

Schneider, BK, Niecknig 2012

Numerical results: $\omega \rightarrow \pi^0 \mu^+ \mu^-$

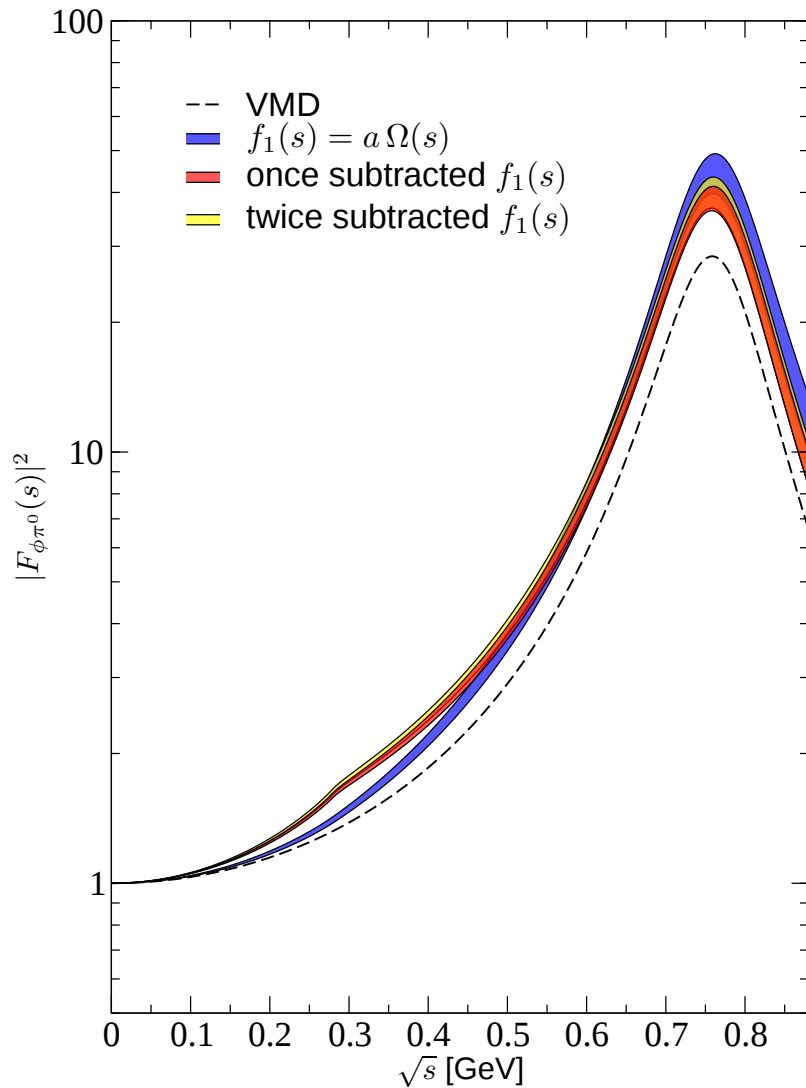


- clear enhancement vs. VMD
- incompatible with data from heavy-ion collisions
NA60 2009, 2011
- $\omega \rightarrow \pi^0 e^+ e^-$ data: no tension (but less precise)
A2 2016

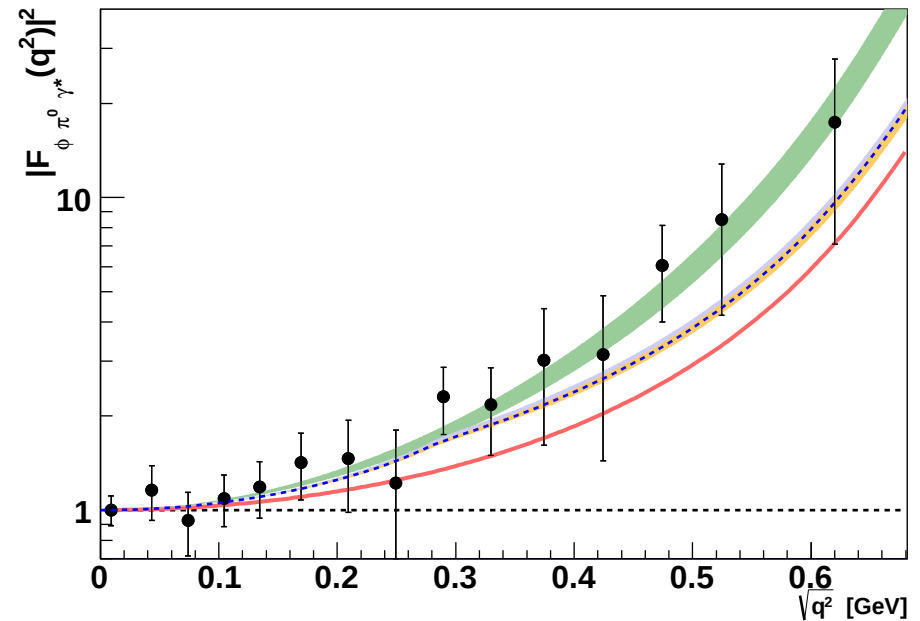
- NA60 data potentially in conflict with unitarity bounds

Ananthanarayan, Caprini, BK 2014, Caprini 2015

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Schneider, BK, Niecknig 2012

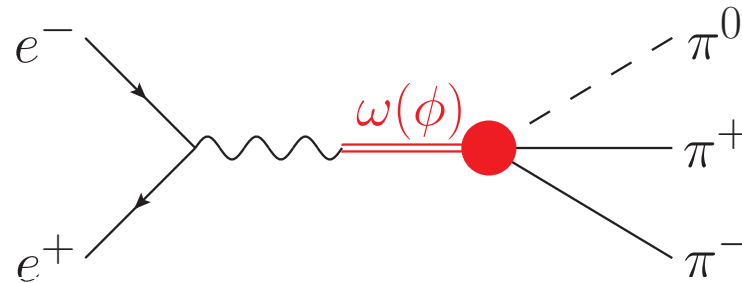


KLOE 2016

- measurement in ρ peak region would be extremely helpful
- $\phi \rightarrow 3\pi$ partial-wave amplitude backed up by experiment

Niecknig, BK, Schneider 2012

One step further: $e^+e^- \rightarrow 3\pi$, $e^+e^- \rightarrow \pi^0\gamma^*$

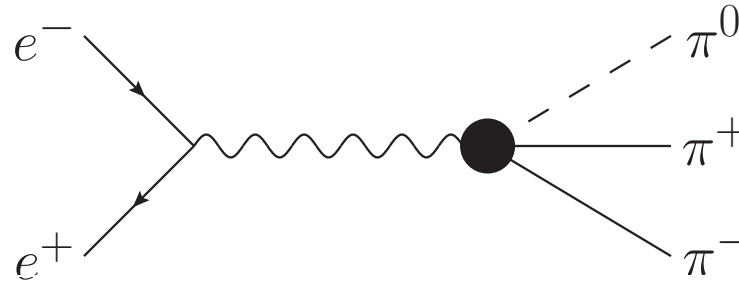


- decay amplitude for $\omega/\phi \rightarrow 3\pi$: $\mathcal{M}_{\omega/\phi} \propto \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$

$$\mathcal{F}(s) = a_{\omega/\phi} \Omega(s) \left\{ 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin \delta_1^1(s') \hat{\mathcal{F}}(s')}{|\Omega(s')|(s' - s)} \right\}$$

$a_{\omega/\phi}$ adjusted to reproduce total width $\omega/\phi \rightarrow 3\pi$

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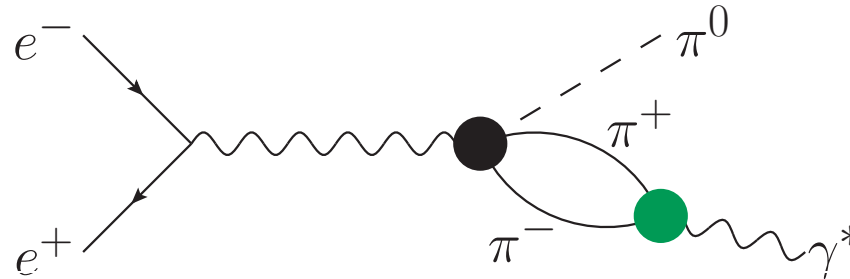
$a_{e^+e^-}(q^2)$ adjusted to reproduce spectrum $e^+e^- \rightarrow 3\pi$

- parameterisation:

$$a_{e^+e^-}(q^2) = \frac{F_{3\pi}}{3} + \beta q^2 + \frac{q^4}{\pi} \int_{\text{thr}}^{\infty} ds' \frac{\text{Im} BW(s')}{s'^2(s' - q^2)}$$

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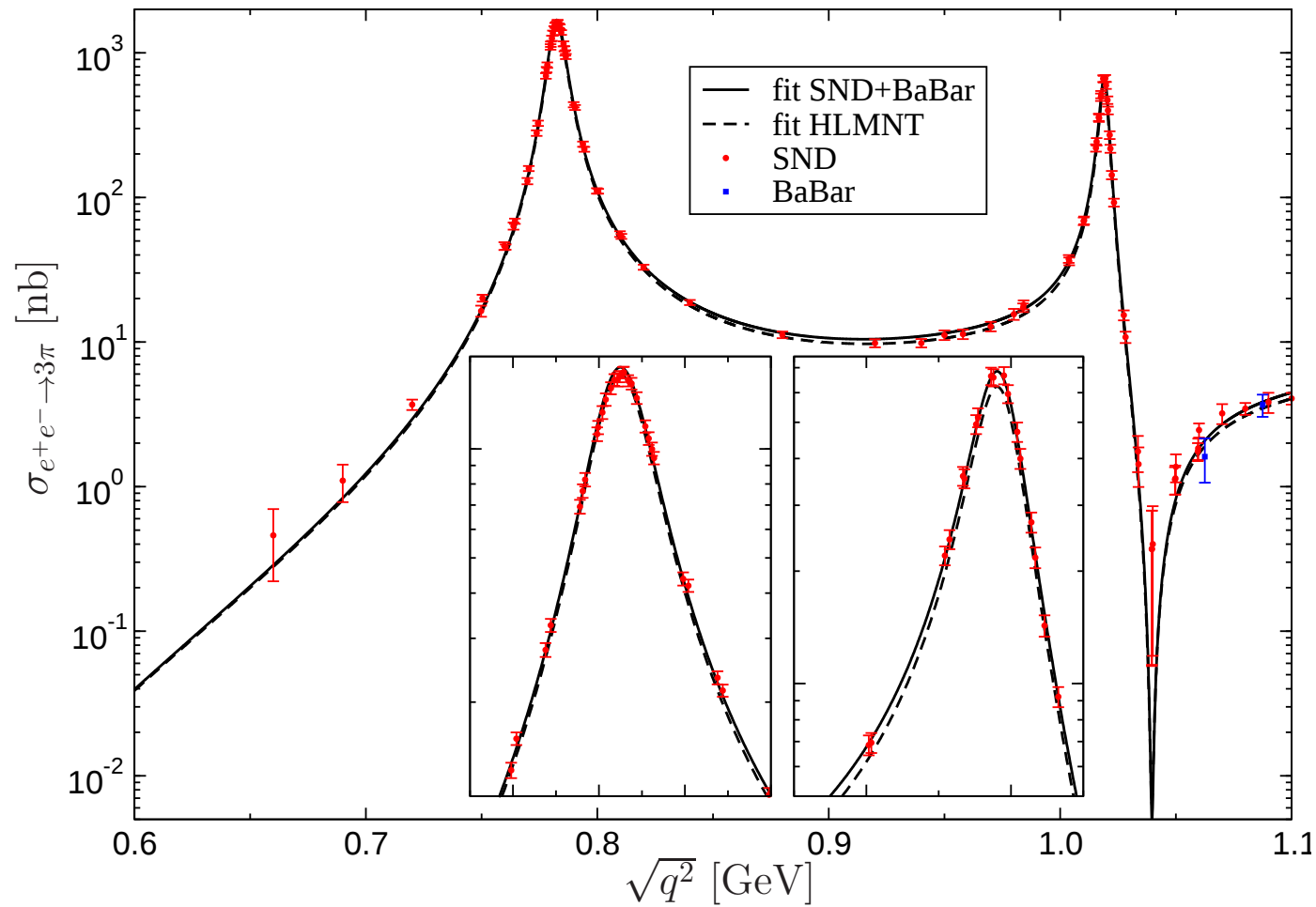
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- fit to $e^+e^- \rightarrow 3\pi$ data $\longrightarrow$ prediction for $e^+e^- \rightarrow \pi^0\gamma^*$

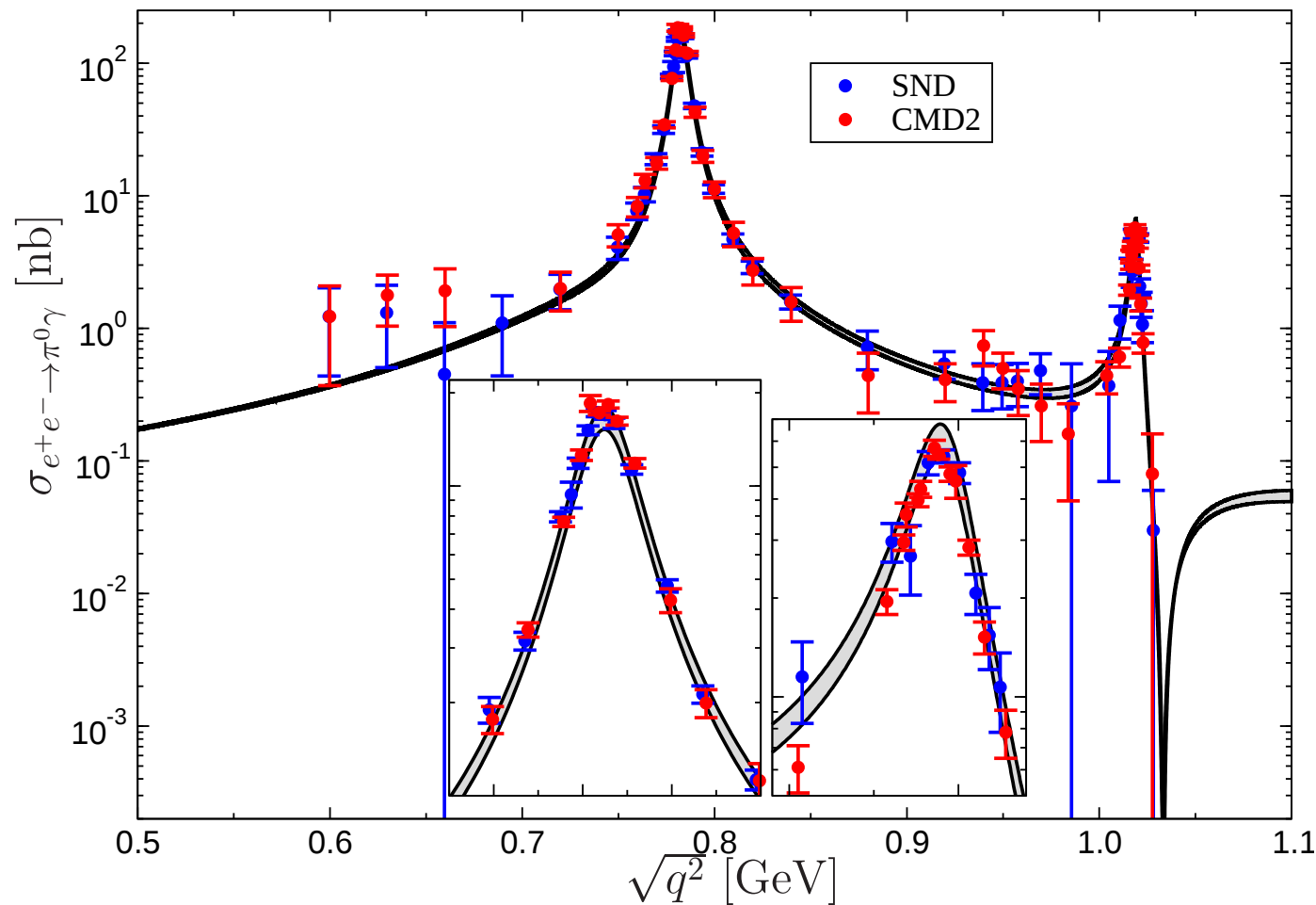
Fit to $e^+e^- \rightarrow 3\pi$ data



Hoferichter, BK, Leupold, Niecknig, Schneider 2014

- one subtraction/normalisation at $q^2 = 0$ fixed by $\gamma \rightarrow 3\pi$
- fitted: ω , ϕ residues, linear subtraction β

Comparison to $e^+e^- \rightarrow \pi^0\gamma$ data



Hoferichter, BK, Leupold, Niecknig, Schneider 2014

- "prediction"—no further parameters adjusted
- data very well reproduced

Extension to spacelike region; slope

- continuation to spacelike region: use another dispersion relation

$$F_{\pi^0\gamma^*\gamma}(q^2, 0) = F_{\pi\gamma\gamma} + \frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\text{Im } F_{\pi^0\gamma^*\gamma}(s', 0)}{s'(s' - q^2)}$$

- sum rule for **slope** $F_{\pi^0\gamma^*\gamma}(q^2, 0) = F_{\pi\gamma\gamma} \left\{ 1 + a_\pi \frac{q^2}{M_{\pi^0}^2} + \mathcal{O}(q^4) \right\}$

$$a_\pi = \frac{M_{\pi^0}^2}{F_{\pi\gamma\gamma}} \times \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2} \text{Im } F_{\pi^0\gamma^*\gamma}(s', 0)$$

$$= (30.7 \pm 0.6) \times 10^{-3}$$

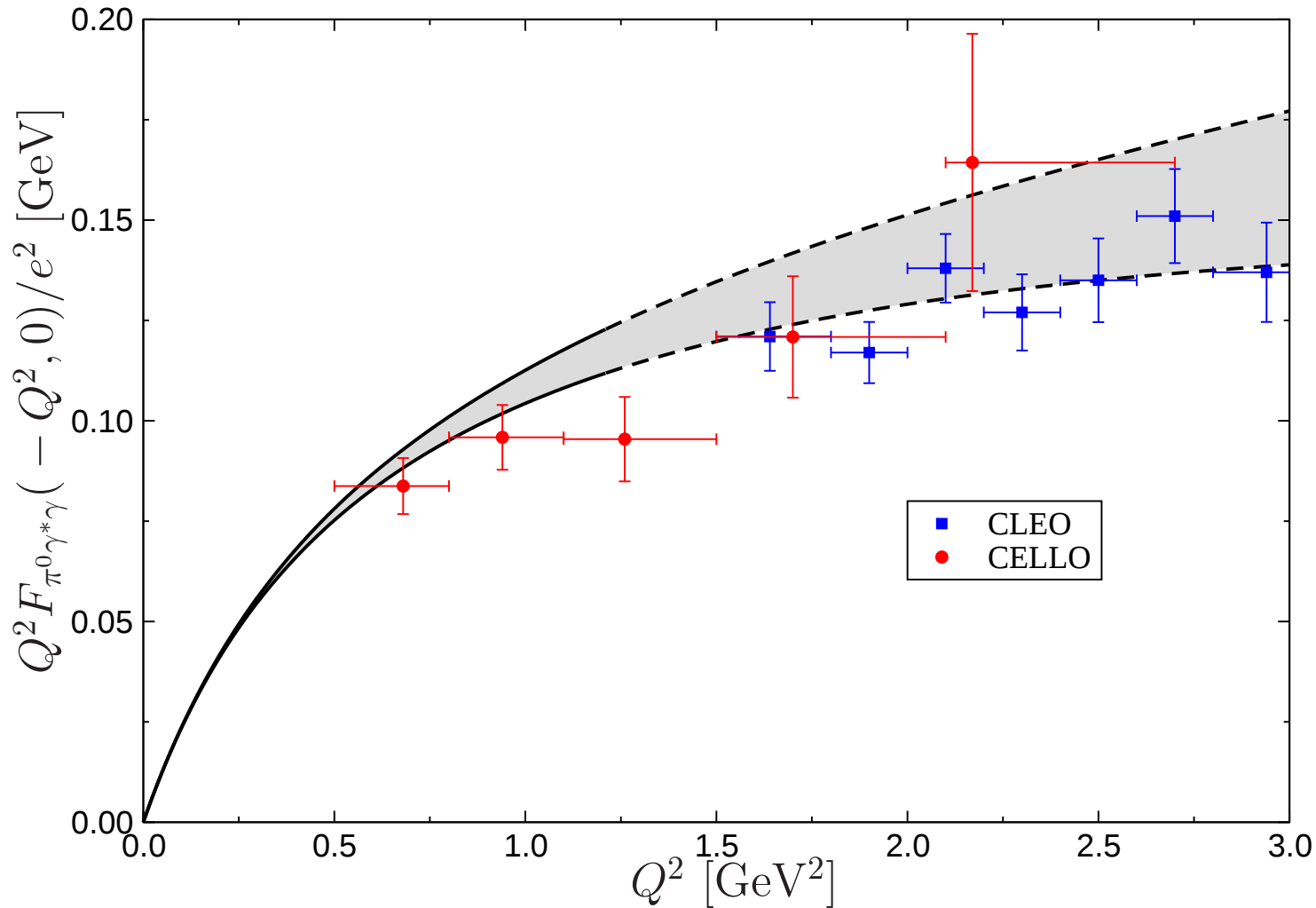
Hoferichter et al. 2014

compare: $a_\pi = (32 \pm 4) \times 10^{-3}$

PDG 2014

- theory error estimate:
 - ▷ $\pi\pi$ phases
 - ▷ cutoff effects in $\gamma^* \rightarrow 3\pi$ partial waves and $[\gamma^* \rightarrow 3\pi] \longrightarrow [\gamma^* \rightarrow \pi^0\gamma]$

Prediction spacelike form factor



Hoferichter, BK, Leupold, Niecknig, Schneider 2014

→ more precise low-energy spacelike data to come

BESIII

Summary / Outlook

Dispersive analyses of π^0 , $\eta^{(\prime)}$ transition form factors:

- high-precision data on

$\eta \rightarrow \pi^+ \pi^- \gamma$ KLOE / $\eta' \rightarrow \pi^+ \pi^- \gamma$ BESIII / $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ var.

allow for high-precision dispersive predictions of π^0 , $\eta^{(\prime)} \rightarrow \gamma^* \gamma$

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Further experimental input:

- Primakoff reactions $\gamma \pi \rightarrow \pi \pi$, $\gamma \pi \rightarrow \pi \eta$ COMPASS
- $\omega \rightarrow 3\pi$ precision Dalitz plot CLAS, BESIII?
- $\omega/\phi \rightarrow \pi^0 \gamma^*$ test **doubly** virtual $F_{\pi^0 \gamma^* \gamma^*}$ with precision
- $e^+ e^- \rightarrow \eta \pi^+ \pi^-$ differential data Xiao et al., in progress

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Theoretical work in progress:

- include **high-energy constraints** L. Bai, PhD thesis
- improved **doubly-virtual** $\eta^{(\prime)}$ ($\leftrightarrow \eta^{(\prime)} \rightarrow 4\pi$) J. Plenter, MSc thesis
- link to lattice M. Niehus, MSc thesis

Spares

Summary: processes and unitarity relations for $\pi^0 \rightarrow \gamma^* \gamma^*$

Colangelo, Hoferichter,
BK, Procura, Stoffer 2014

process	unitarity relations	SC 1	SC 2
	 		$F_{\pi^0 \gamma \gamma}$ $F_{3\pi}$
	 		$\Gamma_{\pi^0 \gamma}$ $\Gamma_{3\pi}$
	 		$\sigma(e^+ e^- \rightarrow \pi^0 \gamma)$ $\sigma(e^+ e^- \rightarrow 3\pi)$ $F_{3\pi}$

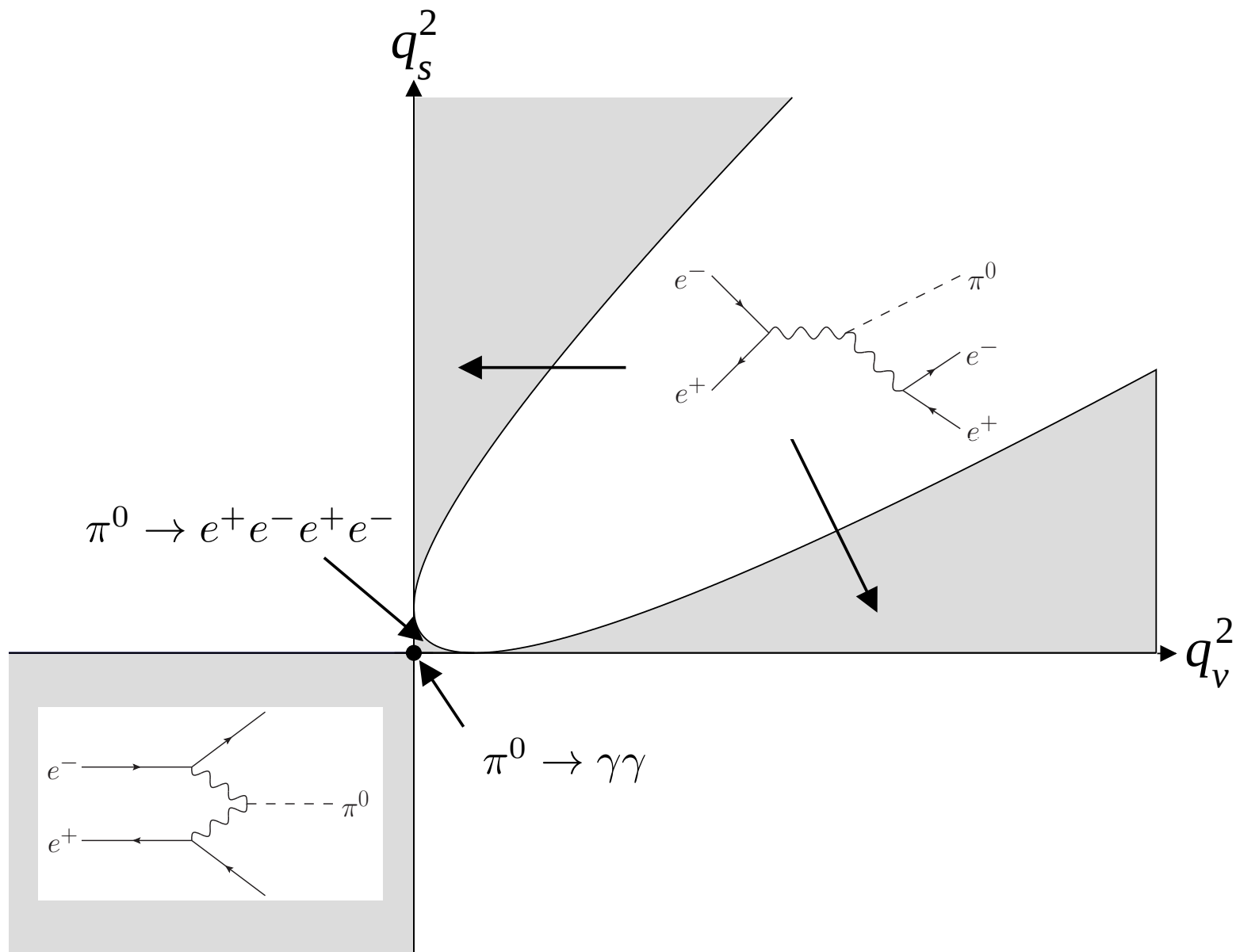
$$\gamma \pi \rightarrow \pi \pi$$

$$\omega \rightarrow 3\pi, \phi \rightarrow 3\pi$$

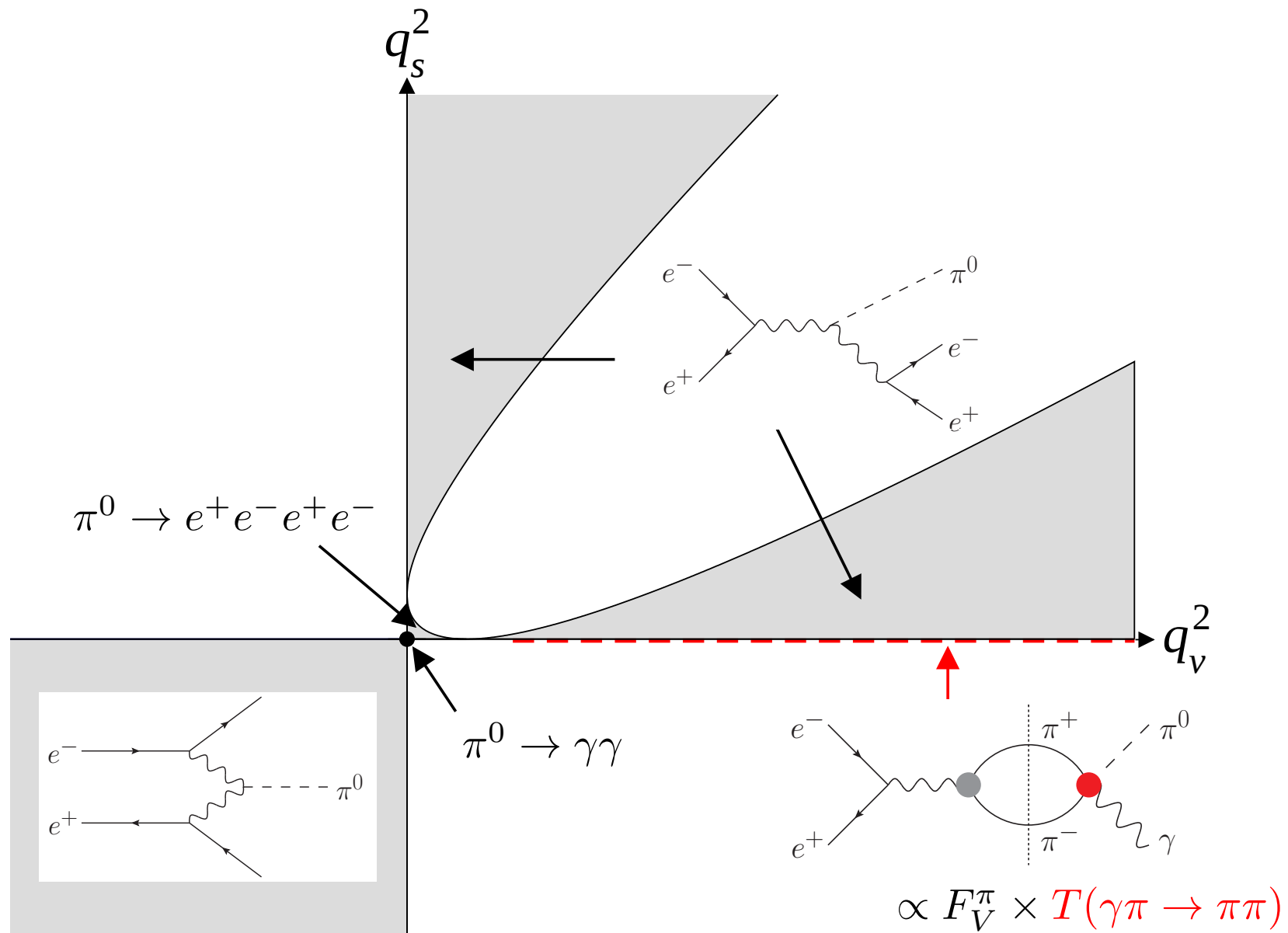
$$\gamma^* \rightarrow 3\pi$$

common theme:
resum $\pi\pi$ rescattering

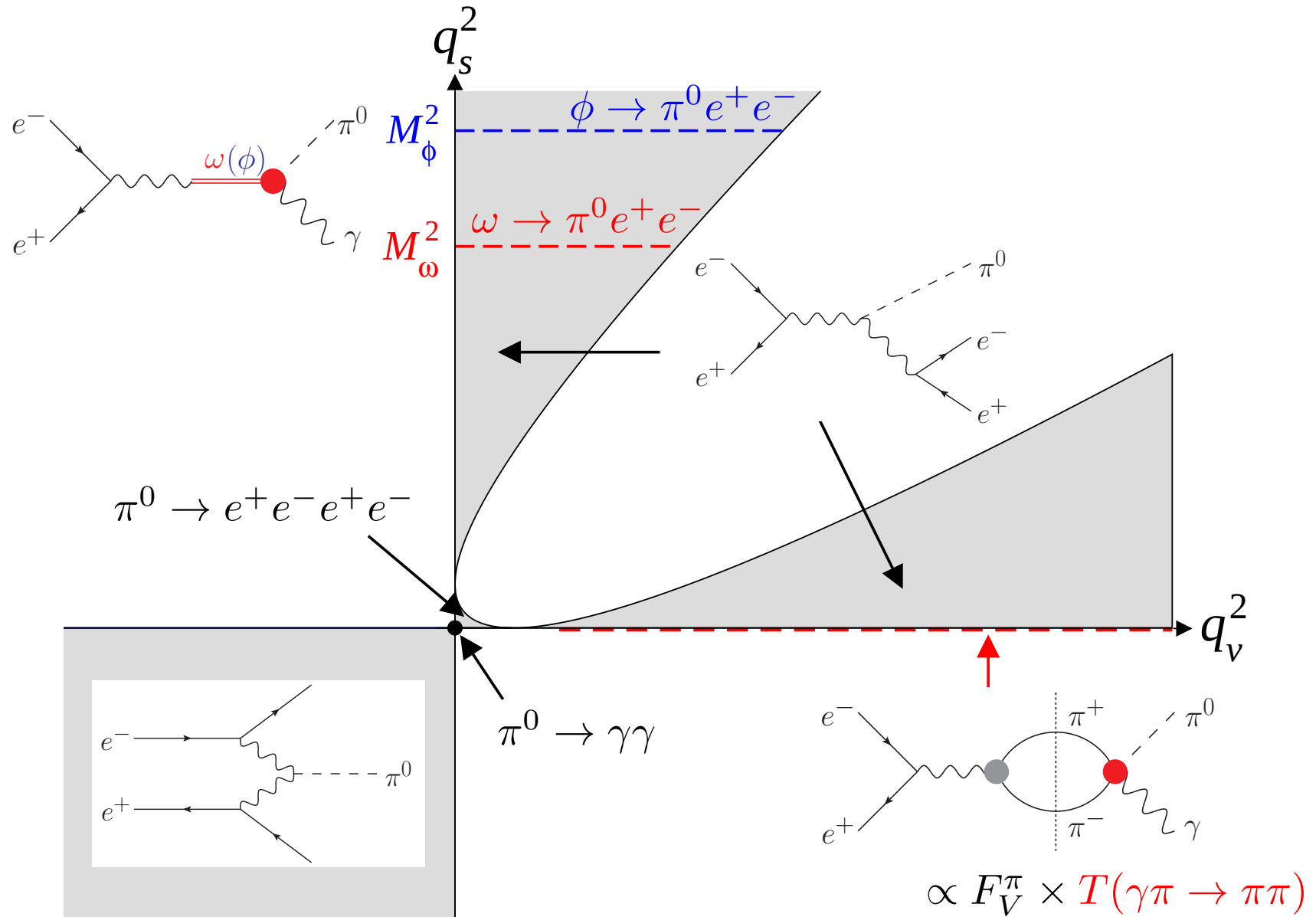
$\pi^0 \rightarrow \gamma^*(q_v^2)\gamma^*(q_s^2)$ transition form factor



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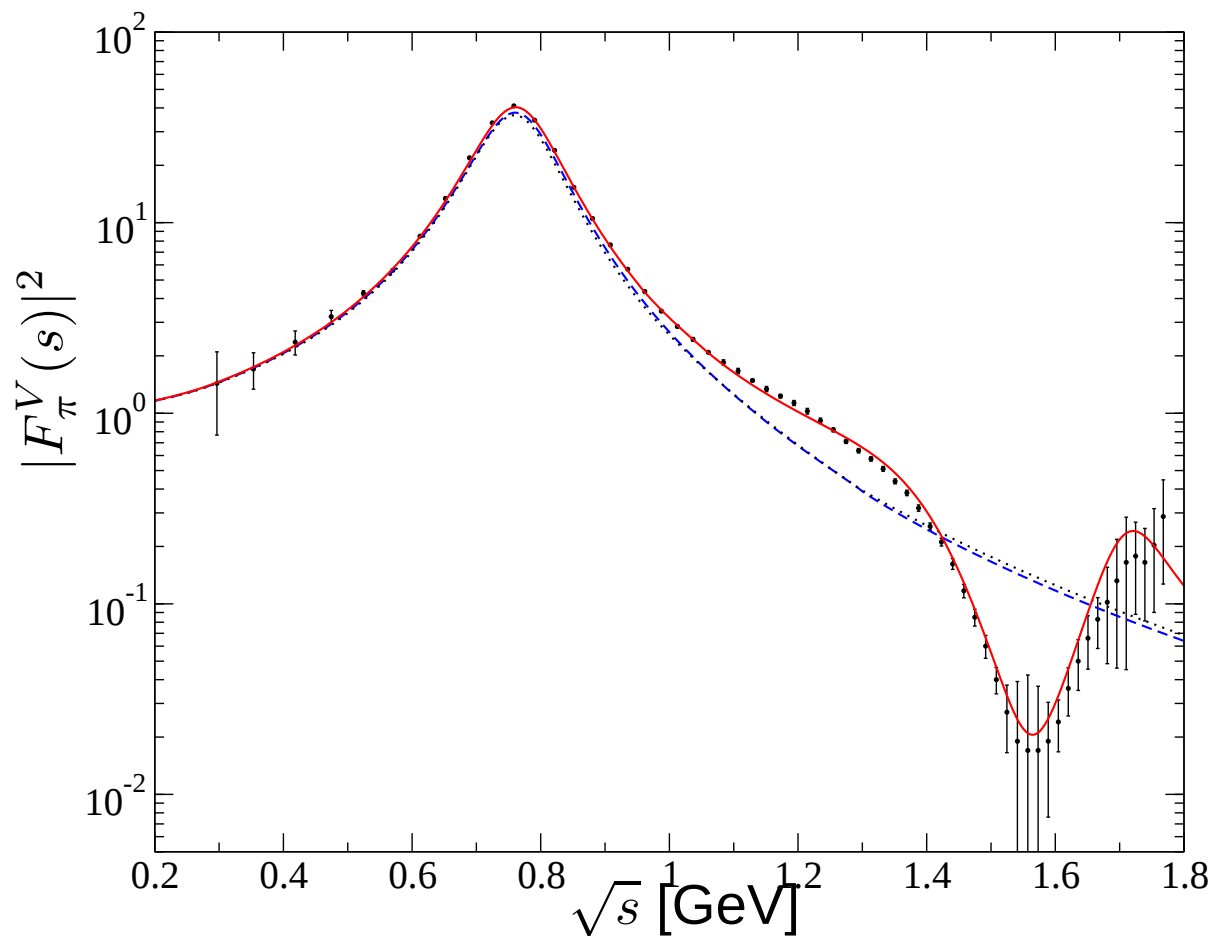
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Pion vector form factor vs. Omnès representation

Data on pion form factor in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

Belle 2008



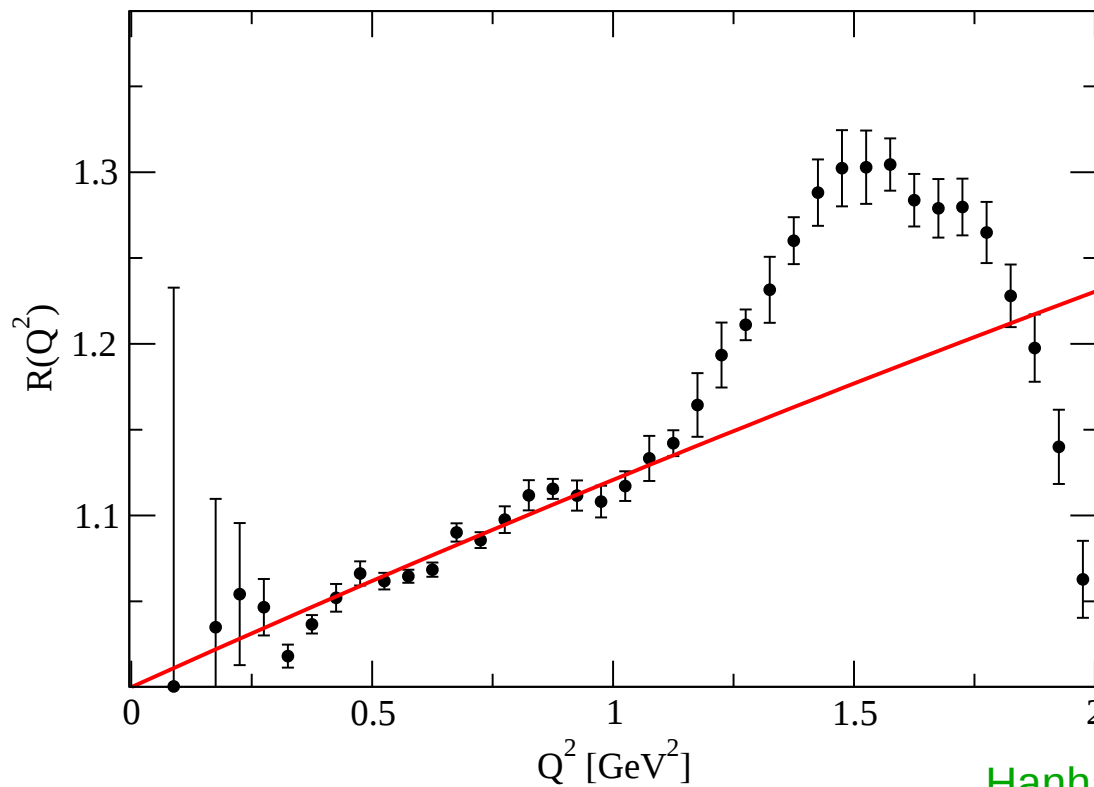
Schneider et al. 2012

Pion vector form factor vs. Omnès representation

Data on pion form factor in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

Belle 2008

- divide $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ form factor by Omnès function:



Hanhart et al. 2013

→ linear below 1 GeV: $F_\pi^V(s) \approx (1 + 0.1 \text{ GeV}^{-2}s)\Omega(s)$

→ above: inelastic resonances ρ' , $\rho'' \dots$

Extension to vector-meson decays: $\omega/\phi \rightarrow 3\pi$

- identical quantum numbers to $\gamma\pi \rightarrow \pi\pi$
- **beyond** ChPT: copious efforts to develop EFT for **vector mesons**
Bijnens et al.; Bruns, Meißner; Lutz, Leupold; Gegelia et al.; Kampf et al. . . .
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+ **constant background term**

KLOE 2003, CMD-2 2006



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KLOE 2003, CMD-2 2006



Problem:

- **unitarity** fixes Im/Re parts
- adding a **contact term** destroys this relation
- reconcile data with dispersion relations?

$\omega/\phi \rightarrow 3\pi$: dispersive solution

- identical quantum numbers to $\gamma\pi \rightarrow \pi\pi$

$$\mathcal{F}(s) = a \Omega(s) \left\{ 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin \delta_1^1(s') \hat{\mathcal{F}}(s')}{|\Omega(s')|(s' - s - i\epsilon)} \right\}$$

$$\hat{\mathcal{F}}(s) = \frac{3}{2} \int_{-1}^1 dz (1 - z^2) \mathcal{F}(t(s, z))$$

→ fix subtraction constant a to partial width(s) $\omega/\phi \rightarrow 3\pi$

$\omega/\phi \rightarrow 3\pi$: dispersive solution

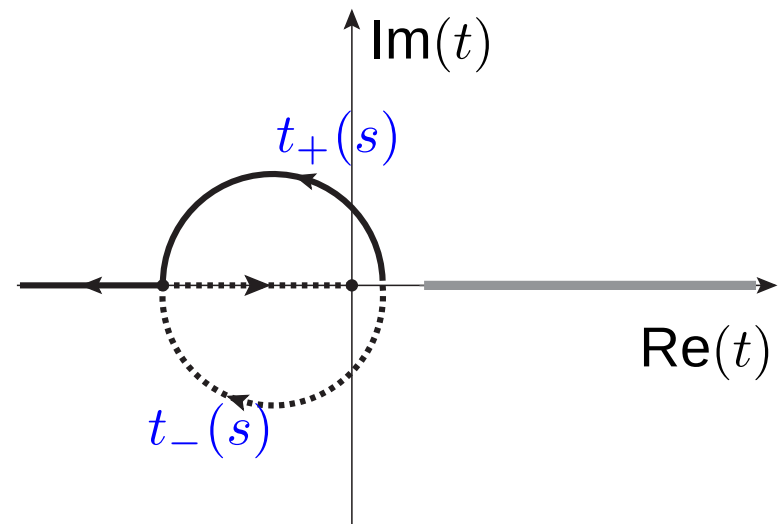
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- complication:**
analytic continuation in
decay mass M_V required
- $M_V < 3M_\pi$:
okay



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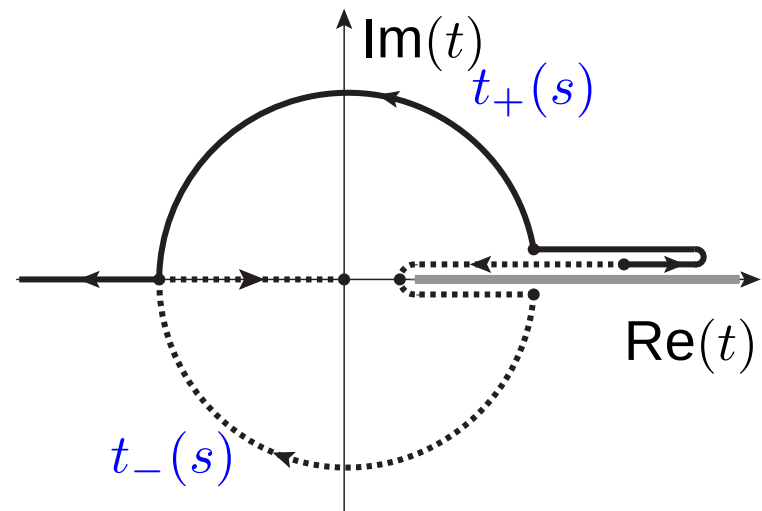
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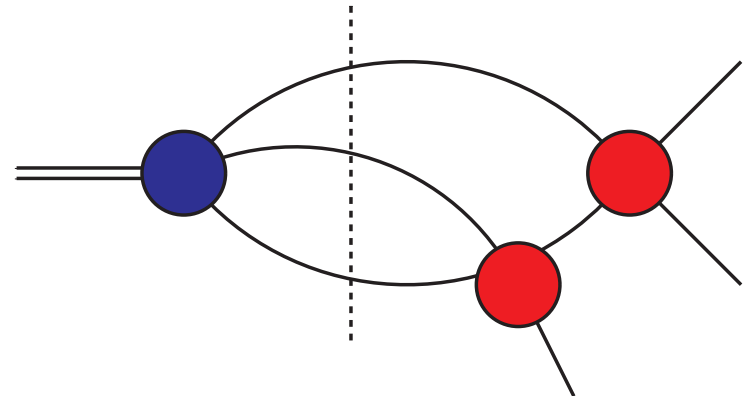
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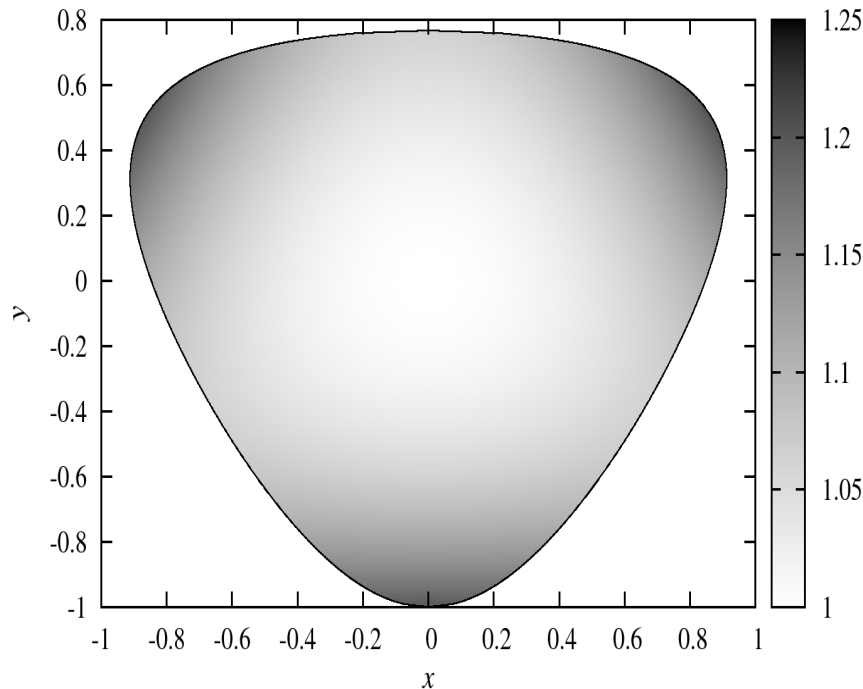
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→ generates **3-particle cuts**



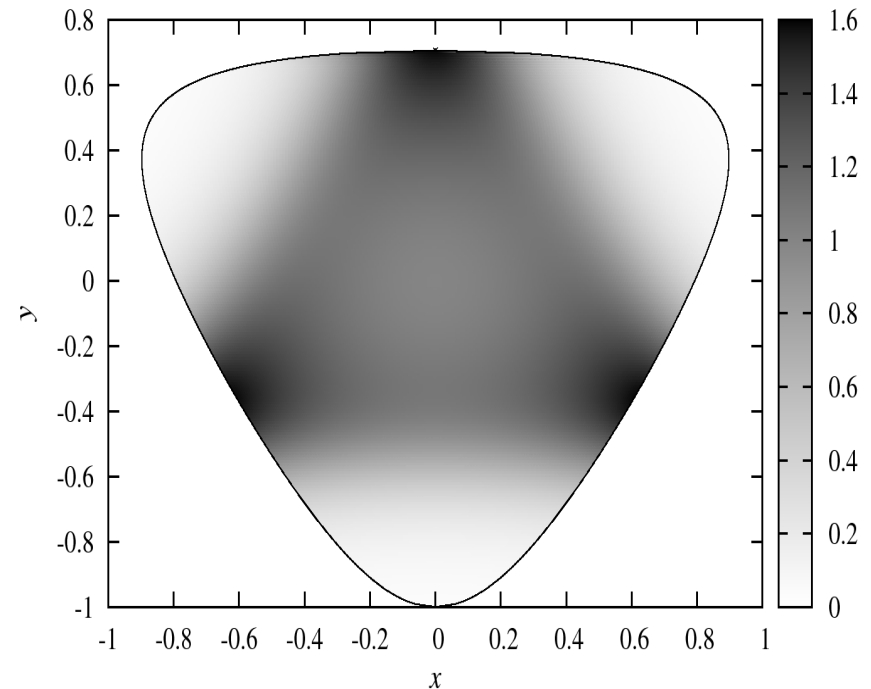
$\omega/\phi \rightarrow 3\pi$ Dalitz plots

- subtraction constant a fixed to partial width
→ normalised Dalitz plot a prediction

$\omega \rightarrow 3\pi$:



$\phi \rightarrow 3\pi$:



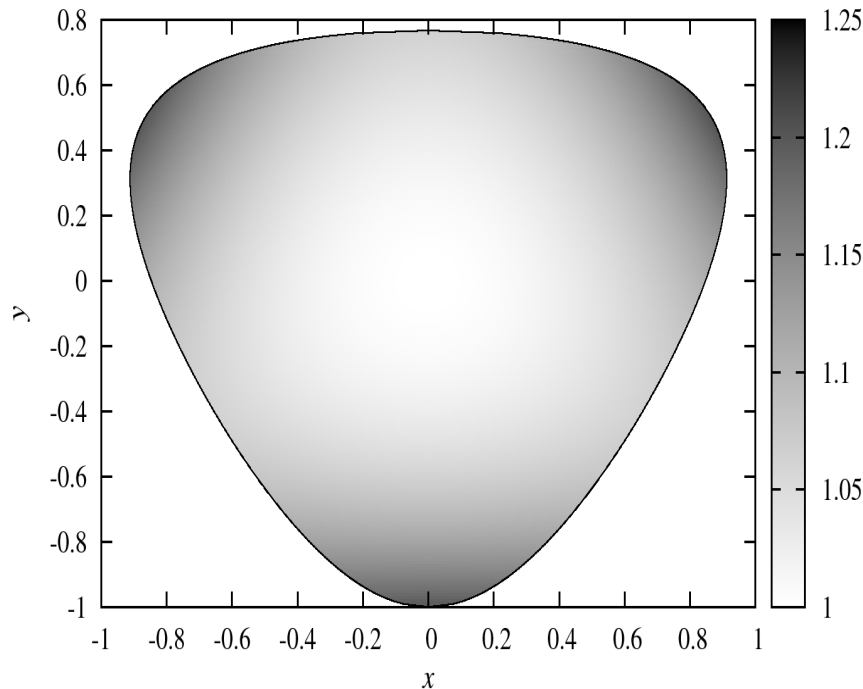
- ω Dalitz plot is relatively smooth
- ϕ Dalitz plot clearly shows ρ resonance bands

Niecknig, BK, Schneider 2012

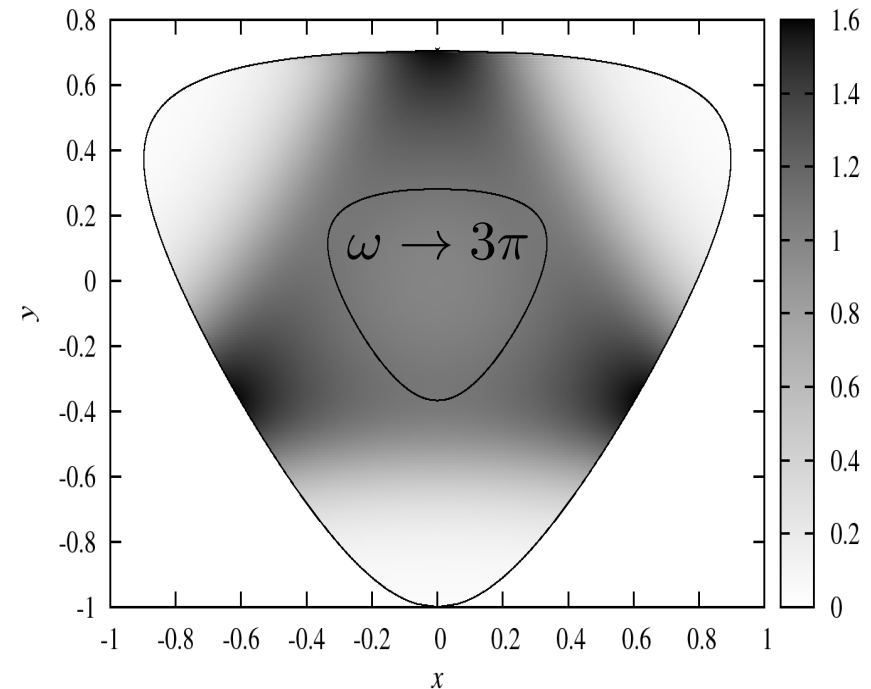
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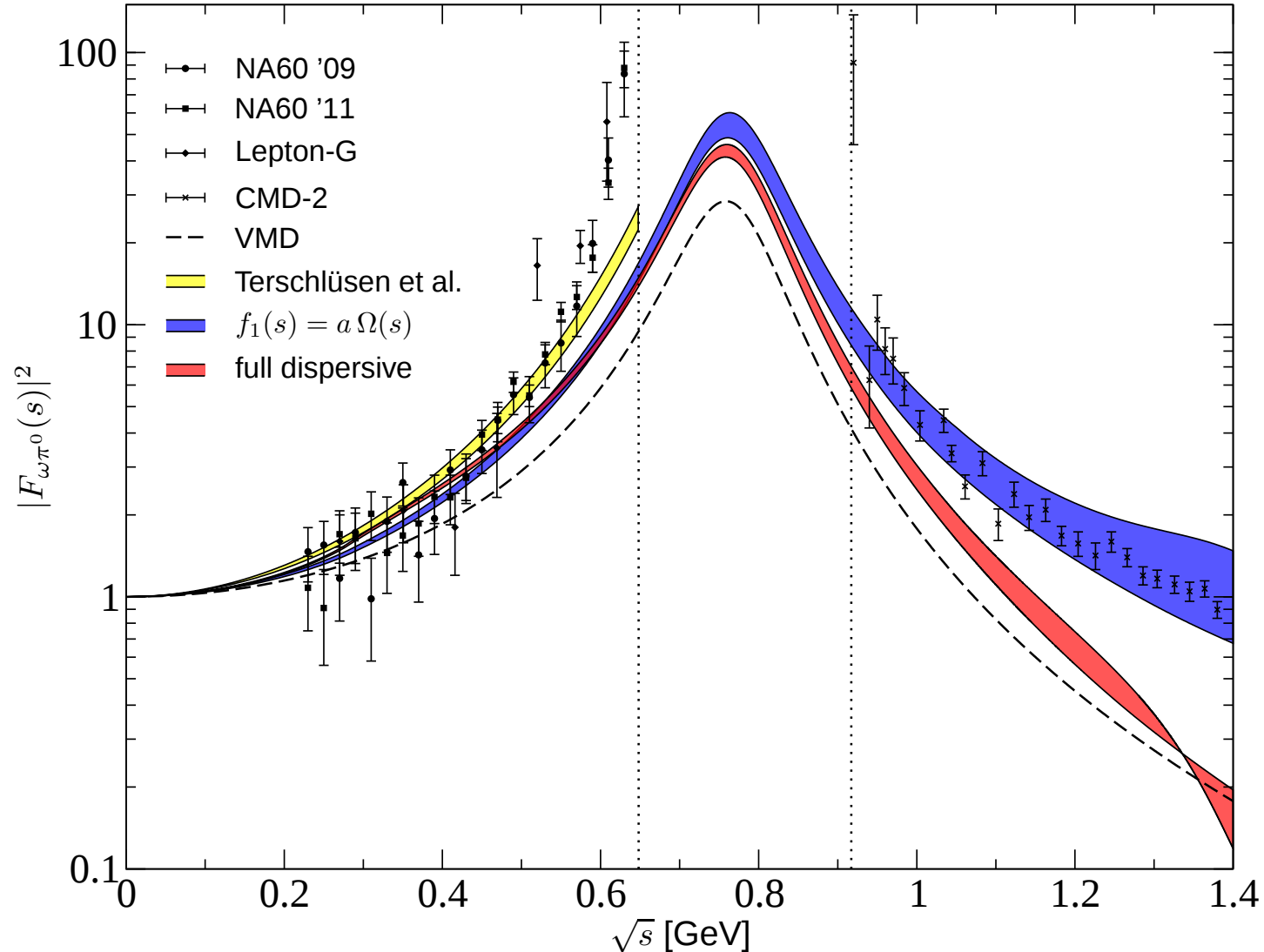
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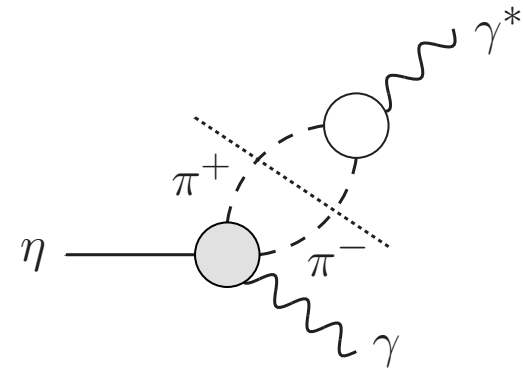
Naive extension to $e^+e^- \rightarrow \pi^0\omega$



- full solution above naive VMD, but still too low
- higher intermediate states ($4\pi / \pi\omega$) more important?

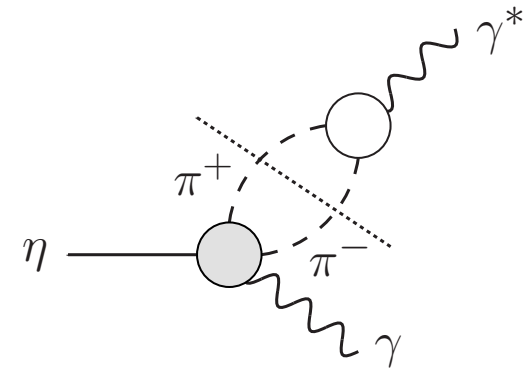
Anomalous decay $\eta \rightarrow \pi^+ \pi^- \gamma$

- $\alpha_{\text{KLOE}} = (1.52 \pm 0.06) \text{ GeV}^{-2}$ **large**
→ implausible to explain through ρ' , $\rho'' \dots$
- for large t , expect $P(t) \rightarrow \text{const.}$ rather
- $\eta \rightarrow \gamma^* \gamma$ **transition form factor**:
→ dispersion integral covers larger energy range



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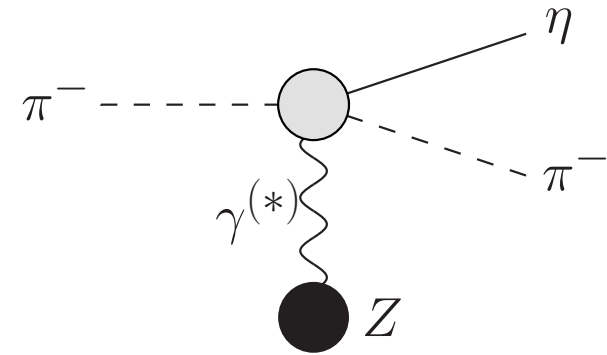
Intriguing observation:

- naive continuation of $\mathcal{F}_{\pi\pi\gamma}^\eta = A(1 + \alpha t)\Omega(t)$ has **zero** at $t = -1/\alpha \approx -0.66 \text{ GeV}^2$
→ test this in **crossed process** $\gamma\pi^- \rightarrow \pi^- \eta$
→ "left-hand cuts" in $\pi\eta$ system?

BK, Plenter 2015

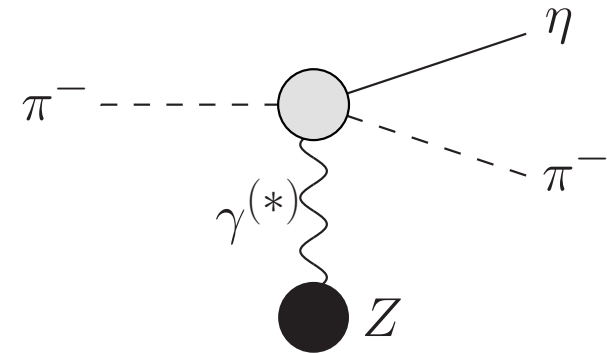
Primakoff reaction $\gamma\pi \rightarrow \pi\eta$

- can be measured in Primakoff reaction COMPASS
- S-wave forbidden
P-wave exotic: $J^{PC} = 1^{-+}$
D-wave $a_2(1320)$ first resonance

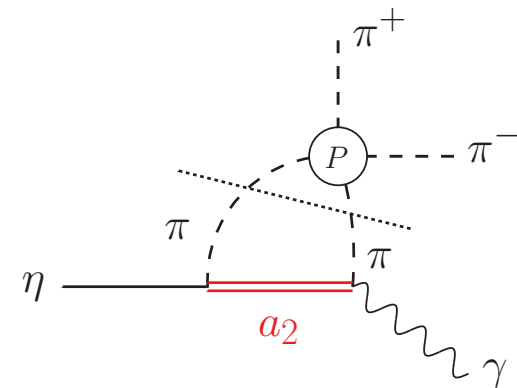
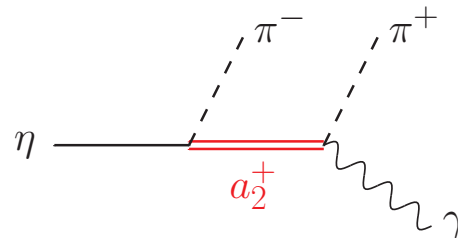
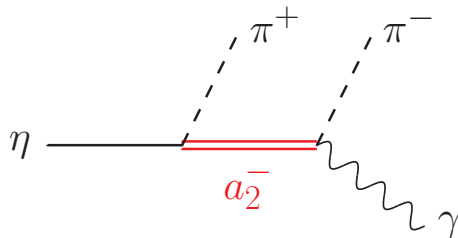


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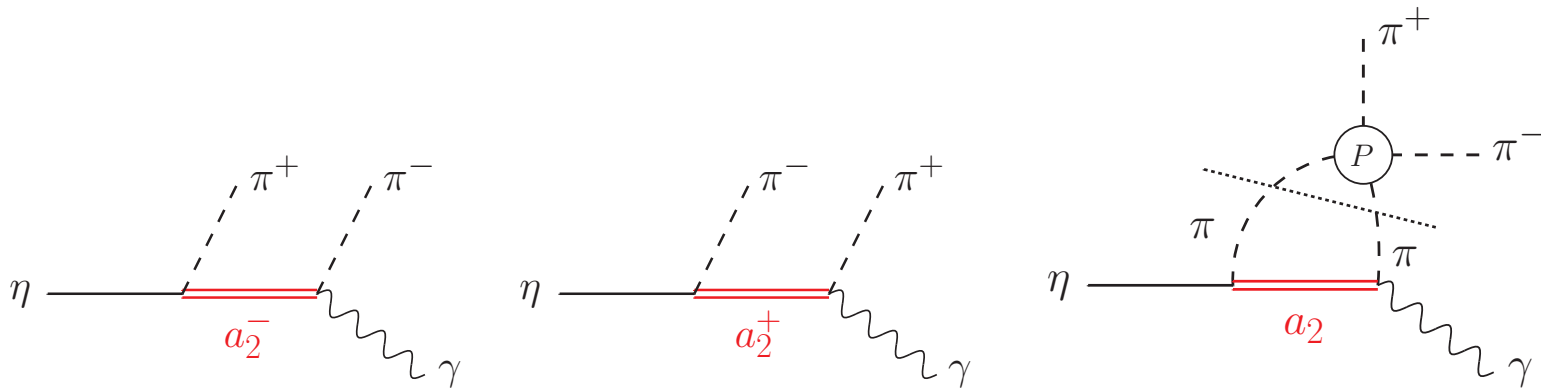
- include a_2 as left-hand cut in decay couplings fixed from $a_2 \rightarrow \pi\eta, \pi\gamma$



- ▷ compatible with decay data?
- ▷ predictions for $\gamma\pi \rightarrow \pi\eta$ cross sections and asymmetries
[$\rightarrow$ spares]

BK, Plenter 2015

Formalism including left-hand cuts



- a_2 + rescattering essential to preserve Watson's theorem
- formally:

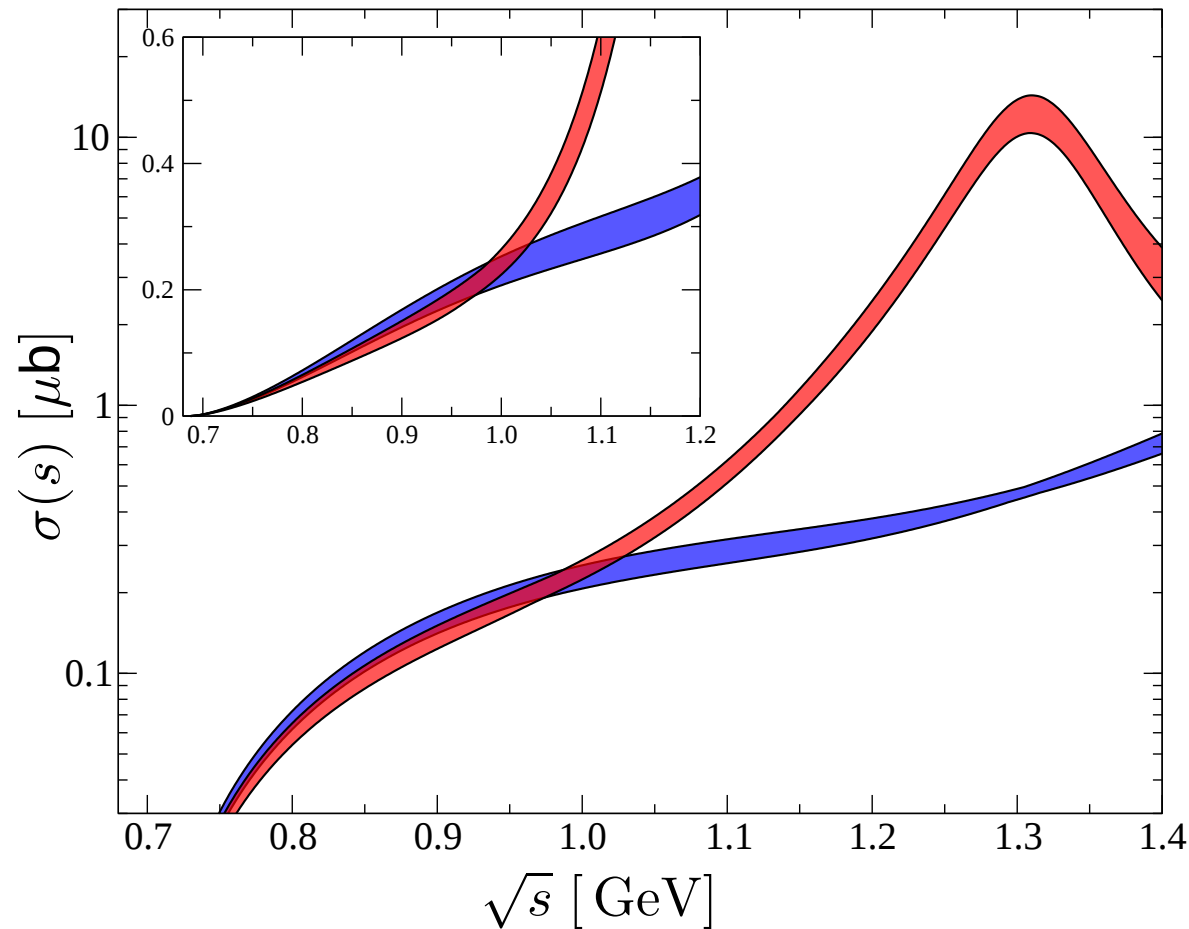
$$\mathcal{F}_{\pi\pi\gamma}^\eta(s, t, u) = \mathcal{F}(t) + \mathcal{G}_{a_2}(s, t, u) + \mathcal{G}_{a_2}(u, t, s)$$

$$\mathcal{F}(t) = \Omega(t) \left\{ A(1 + \alpha t) + \frac{t^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dx}{x^2} \frac{\sin \delta(x) \hat{\mathcal{G}}(x)}{|\Omega(x)|(x - t)} \right\}$$

$\hat{\mathcal{G}}$: t -channel P-wave projection of a_2 exchange graphs

- re-fit subtraction constants A , α

Total cross section $\gamma\pi \rightarrow \pi\eta$



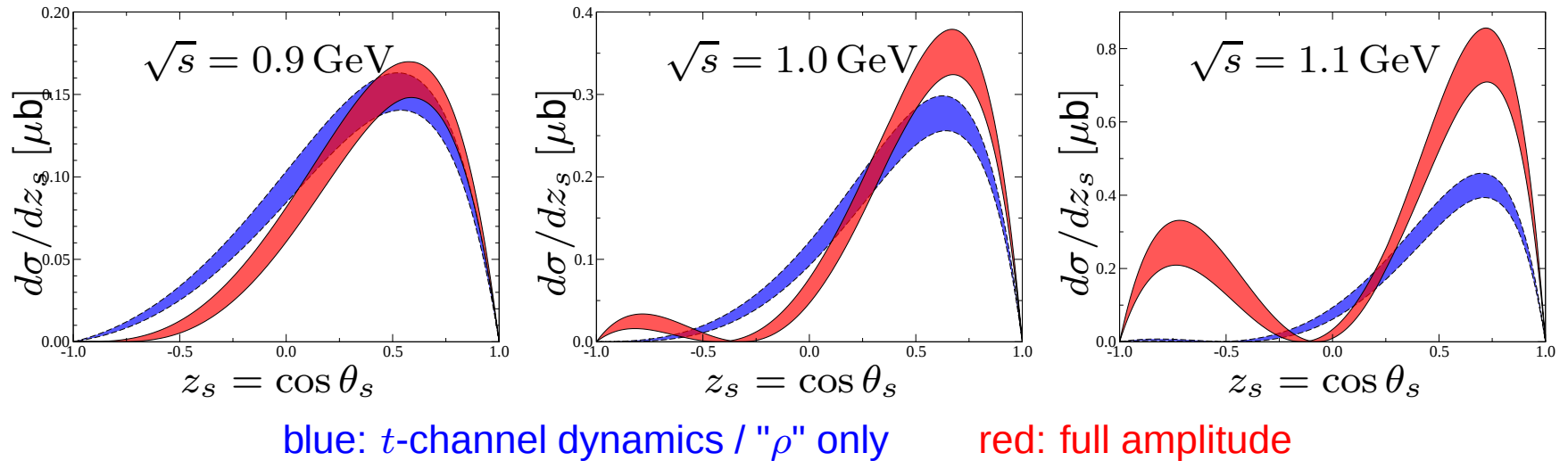
blue: t -channel dynamics / " ρ " only

red: full amplitude

- t -channel dynamics dominate below $\sqrt{s} \approx 1$ GeV
- uncertainty bands: $\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma)$, α , a_2 couplings [BK, Plenter 2015](#)

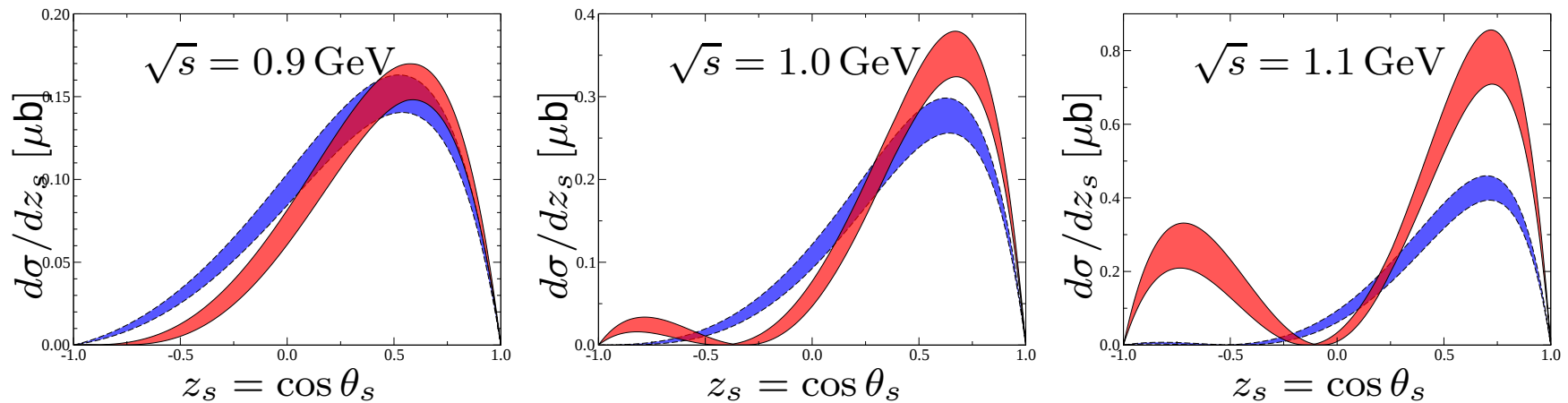
Differential cross sections $\gamma\pi \rightarrow \pi\eta$

- amplitude **zero** visible in differential cross sections:



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red: full amplitude

- strong P-D-wave interference
- can be expressed as **forward-backward asymmetry**

$$A_{\text{FB}} = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma_{\text{total}}}$$

