FLAVOUR PHYSICS and HEAVY QUARKS

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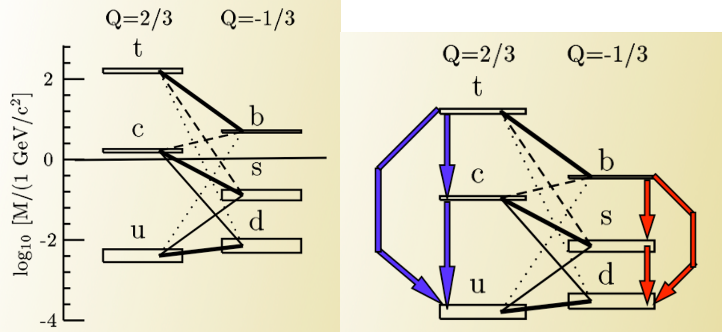
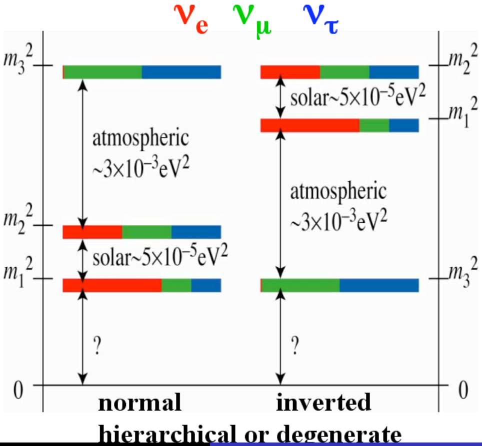
Preliminary Remarks

- Flavour Physics:
  Transitions between different kinds of Quarks



- Its all about weak interactions ...
- Strong interactions as a "background"

- Likewise for Leptons, but
- no strong interactions here
- Neutrinos hard to detect
  $\to$ Flavour Identification



More reading ...

- R. Fleischer: **Flavour Physics and CP Violation**
  Lectures given at European School of High-Energy Physics 2005
  hep-ph/0608010 → Non-leptonics and CP
- A. Buras: **Flavor physics and CP violation**
  Lectures given at European School of High-Energy Physics 2004
  hep-ph/0505175 → rare FCNC decays
- A. Buras: **Minimal flavor violation**
  Lectures given at 43rd Cracow School of Theoretical Physics 2003
  hep-ph/0310208 → MFV and New Physics

- Y. Nir: **Probing new physics with flavor physics**
  Lectures given at 2nd Joint Fermilab-CERN Hadron Collider Physics Summer School 2007
  arXiv:0708.1872 [hep-ph] $\rightarrow$ Mainly New Physics
- A. Bevan, B. Golob, T. Mannel, S. Prell, B. Yabsley (eds.) **The Physics of the *B* Factories**
  Eur.Phys.J. **C74** (2014) 3026, (926 pages)

Outline of the course

- Lecture 1: Flavour in the Standard Model
- Lecture 2: Theoretical Tools and Phenomenology
- Lecture 3: Flavour beyond the Standard Model

Lecture 1
Flavour in the Standard Model

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Outline of Lecture 1

1. Quarks in the SM: $SU(2)_L \times U(1)_Y$
   - Symmetries and Quantum Numbers
   - Quark Mixing and CKM Matrix
2. Leptons In the Standard Model
   - Assignement of Quantum Numbers
   - See Saw Mechanism
   - PMNS Matrix
3. Peculiarities of Flavour in the Standard Model
   - Peculiarities of SM CP / Flavour

Gauge Structure of the Standard Model

I assume a few things to be known:

- The Standard Model is a gauge theory based on
  $SU(3)_{QCD} \otimes SU(2)_{Weak} \otimes U(1)_{Hypercharge}$
- Eight gluons, three weak gauge bosons, one photon
- Matter (quarks and leptons):
  Multiplets of the gauge group $\to$ Quantum numbers
- Spontaneous Symmetry Breaking:
  Introduction of scalar fields
- Massless Goldstone Modes:
  Higgs Mechanism:
  $\phi \to$ longitudinal modes of gauge bosons: $\phi \sim \partial_\mu W^\mu$

Matter Fields: Quarks

- Left Handed Quarks:
  $SU(3)_C$ Triplets, $SU(2)_L$ Doublets

$$Q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \; Q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \; Q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

  $SU(2)_L$ will be gauged

- Right Handed Quarks:
  $SU(3)_C$ Triplets, $SU(2)_R$ Doublets

$$q_1 = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \; q_2 = \begin{pmatrix} c_R \\ s_R \end{pmatrix} \; q_3 = \begin{pmatrix} t_R \\ b_R \end{pmatrix}$$

  $SU(2)_R$ introduced “artificially”

Quantum Numbers

- Hypercharge

$$Y = T_{3,R} + \frac{1}{2}(B - L)$$

- Charge

$$q = T_{3,L} + Y = T_{3,L} + T_{3,R} + \frac{1}{2}(B - L)$$

Higgs Fields: Standard Model

- Single $SU(2)$ Doublett: Two Complex Fields

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

- Charge Conjugate Field is also an $SU(2)$ Doublett

$$\widetilde{\Phi} = (i\tau_2)\Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- = -\phi_+^* \end{pmatrix}$$

- It is useful to gather these into a 2×2 matrix

$$H = \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_- & \phi_0 \end{pmatrix}$$

- Transformation Properties: $L \in SU(2)_L$:

$$\Phi \to L\Phi \qquad \widetilde{\Phi} \to L\widetilde{\Phi}$$

- Transformation Properties: $R \in SU(2)_R$:

$$\begin{pmatrix} \phi_0 \\ \phi_- \end{pmatrix} \to R \begin{pmatrix} \phi_0 \\ \phi_- \end{pmatrix} \qquad \begin{pmatrix} \phi_+ \\ -\phi_0^* \end{pmatrix} \to R \begin{pmatrix} \phi_+ \\ -\phi_0^* \end{pmatrix}$$

- In total:

$$H \to LHR^\dagger \qquad (\text{remember } \; Q \to LQ \quad q \to Rq)$$

- Hypercharges

$$Y\Phi = -\Phi \qquad Y\widetilde{\Phi} = \widetilde{\Phi} \qquad YH = -HT_{3,R}$$

Gauge Interactions

- $SU(3)_{color}$ is gauged (not relevant for us now)
- $SU(2)_L$ is gauged Three W_a^μ Bosons
- Hypercharge is gauged One B^μ Boson
- Recipe: Replace the ordinary derivative in the kinetic terms by the covariant one

$$\partial^\mu \to D^\mu = \partial^\mu - igT_{L,a}W_a^\mu - iYB^\mu$$
$$+\text{QCD interactions}$$

- Weinberg rotation between W_3^μ and B^μ $\cdots$
- I assume you have heard the rest of the story ...
- This is not relevant for the phenomenon of masses and mixing !

Structure of the Standard Model

- Start out from an $SU(2)_L \times SU(2)_R$ symmetric case:
- Kinetic Term for Quarks and Higgs (i: Generation)

$$\mathcal{L}_{kin} = \sum_i \left[\bar{Q}_i \not{\partial} Q_i + \bar{q}_i \not{\partial} q_i \right] + \frac{1}{2} \mathrm{Tr} \left[(\partial_\mu H)^\dagger (\partial^\mu H) \right]$$

- Potential for the Higgs field

$$V = V(H) = V(\mathrm{Tr}\left[H^\dagger H\right])$$

- Interaction between Quarks and Higgs

$$\mathcal{L}_I = -\sum_{ij} y_{ij} \bar{Q}_i H q_j + \text{ h.c.}$$

- y_{ij} can be made diagonal: Any Matrix y can be diagonalized by a Bi-Unitary Transformation:

$$y = U^\dagger y_{diag} W$$

- Thus

$$\mathcal{L}_I = -\sum_{ijk} \bar{Q}_i (U^\dagger)_{ik} y_k W_{kj} H q_j + \text{ h.c.}$$

- Rotation of Q_i and q_j:

$$Q' = UQ \quad q' = Wq$$

- This has no effect on the kinetic term: $y_{ij} = y_i \delta_{ij}$ is the general case!

$$\mathcal{L}_I = -\sum_i y_i \bar{Q}_i H q_i + \text{ h.c.}$$

Sponaneous Symmetry Breaking

- The Higgs Potential is (Renormalizability):

$$V = \kappa \left(\mathrm{Tr}\left[H^\dagger H\right]\right) + \lambda \left(\mathrm{Tr}\left[H^\dagger H\right]\right)^2$$

- For $\kappa < 0$ we have SSB:
  H acquires a Vacuum Expectation Value (VEV)

$$\mathrm{Tr}\left[\langle H^\dagger \rangle \langle H \rangle\right] = -\frac{\kappa}{2\lambda} > 0$$

- Choice of the VEV

$$< \mathrm{Re}\phi_0 >= v \text{ or } < H >= v\, 1_{2\times 2}$$

- Three massless fields: ϕ_+, ϕ_-, $\text{Im}\phi_0$: Goldstone Bosons
- $\phi_0 \to v + \phi_0'$: One massive field
- Higgs Mechanism: The massless scalars become the longitudinal modes of the massive vector bosons:
  - $\phi_\pm \sim \partial^\mu W_\mu^\pm$
  - $\text{Im}\phi_0 \sim \partial^\mu Z_\mu$
- ϕ_0': Physical Higgs Boson

- The Quarks become massive:

$$\mathcal{L}_I = -\sum_i y_i v \bar{Q}_i q_i + \text{ h.c. } + \cdots$$

- We have $\bar{Q}_1 q_1 = \bar{u}_L u_R + \bar{d}_L d_R$ etc.
- Thus

$$\mathcal{L}_{mass} = -m_u(\bar{u}u + \bar{d}d) - m_c(\bar{c}c + \bar{s}s) - m_t(\bar{t}t + \bar{b}b)$$

- This is not (yet) what we want ...
- We still have too much symmetry!

Custodial $SU(2)$

- Symmetry of the Higgs Sector in the Standard Model:

$$SU(2)_L \otimes SU(2)_R \xrightarrow{SSB} SU(2)_{L+R} = SU(2)_C$$

- Note that we cannot have explicit breaking of $SU(2)_R$ in the Higgs sector:

$$\mathrm{Tr}\left[H\tau_i H^\dagger\right] = 0$$

- $SU(2)_C$: Custodial Symmetry!
  $\to$ Extra Symmetry in the Higgs sector !
- This is more than needed: Only $U(1)_Y$ is needed
- $U(1)_Y$ will be related to the τ_3 direction of $SU(2)_R$

- Consequences of $SU(2)_C$:
  - Relation between charged and neutral currents: ρ parameter
  - Masses of $W^{\pm}$ and of Z^0 are equal
  - Up- and Down-type quark masses are equal in each family
  - No mixing occurs among the families
- $SU(2)_C$ is broken by:
  - Yukawa Couplings
  - Gauging only the Hypercharge

$$Y = T_3^{(R)} + \frac{1}{2}(B - L)$$

Breaking $SU(2)_C$: Yukawa Couplings

- Explicit breaking of $SU(2)_C$ by Yukawa Couplings:

$$\mathcal{L}'_I = -\sum_{ij} y'_{ij} \bar{Q}_i H (2T_{3,R}) q_j + \text{ h.c.}$$

- Effect of this term:
  - Introduces a splitting between up- and down quark masses
  - Introduces mixing between different families
  - Affects the ρ parameter
- Total Yukawa Coupling term:

$$\mathcal{L}_I + \mathcal{L}'_I = -\sum_{ij} \bar{Q}_i H (y_i \delta_{ij} + 2T_{3,R} y'_{ij}) q_j + \text{ h.c.}$$

Quark Mass Matrices

- Use the projections

$$P_\pm = \frac{1}{2} \pm T_{3,R} \quad \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Up quark Yukawa couplings:

$$\mathcal{L}^u_{mass} = -\sum_{ij} \bar{Q}_i H (y_i \delta_{ij} + y'_{ij}) P_+ q_j + \text{ h.c.}$$

- Down quark Yukawa couplings:

$$\mathcal{L}^d_{mass} = -\sum_{ij} \bar{Q}_i H (y_i \delta_{ij} - y'_{ij}) P_- q_j + \text{ h.c.}$$

- $\rightarrow$ mass terms, once $\mathrm{Re}\phi_0 \rightarrow v$

- More compact notation

$$\mathcal{U}_{L/R} = \begin{bmatrix} u_{L/R} \\ c_{L/R} \\ t_{L/R} \end{bmatrix} \quad \mathcal{D}_{L/R} = \begin{bmatrix} d_{L/R} \\ s_{L/R} \\ b_{L/R} \end{bmatrix}$$

- Mass Term for Up-type quarks

$$\mathcal{L}^u_{mass} = -v \, \bar{\mathcal{U}}_L Y^u \mathcal{U}_R + \text{ h.c.}$$

with $Y^u = (y + y')$

- Mass Term for down-type quarks

$$\mathcal{L}^d_{mass} = -v \, \bar{\mathcal{D}}_L Y^d \mathcal{D}_R + \text{ h.c.}$$

with $Y^d = (y - y')$

- Mass matrices:

$$\mathcal{M}^u = v\,Y^u \qquad \mathcal{M}^d = v\,Y^d$$

- In general non-diagonal: Diagonalization by a bi-unitary transformation:

$$\mathcal{M} = U^\dagger \mathcal{M}_{diag} W$$

- New basis for the quark fields

$$\mathcal{L}^u_{mass} = -\bar{\mathcal{U}}_L U^{u,\dagger} \mathcal{M}^u_{diag} W^u \mathcal{U}_R + \text{ h.c.}$$

and

$$\mathcal{L}^d_{mass} = -\bar{\mathcal{D}}_L U^{d,\dagger} \mathcal{M}^d_{diag} W^d \mathcal{D}_R + \text{ h.c.}$$

Quark Mixing: The CKM Matrix

- Effect of the basis transformation:
  - Mass matrices become diagonal
  - Interaction with $\mathrm{Re}\,\phi_0$ (= Physical Higgs Boson) becomes diagonal !
  - Interaction with $\mathrm{Im}\,\phi_0$ (= Z_0) becomes diagonal !

$$\begin{aligned}\mathcal{L}_{\mathrm{Re}\,\phi_0} &= -\mathrm{Re}\,\phi_0[\mathcal{U}_L Y^u \mathcal{U}_R + \mathcal{D}_L Y^d \mathcal{D}_R] \\ \mathcal{L}_{\mathrm{Im}\,\phi_0} &= -\mathrm{Im}\,\phi_0[\mathcal{U}_L Y^u \mathcal{U}_R - \mathcal{D}_L Y^d \mathcal{D}_R]\end{aligned}$$

- NO FLAVOUR CHANGING NEUTRAL CURRENTS (at tree level in the Standard Model)
- $\rightarrow$ GIM Mechanism

- Effect on the charged current ONLY:
  Interaction with ϕ_-:

$$\begin{aligned}&\sum_{ij} \bar{Q}_i (y_i \delta_{ij} + y'_{ij}) \phi_- \tau_- P_+ q_j + \text{ h.c.} \\ &= \mathcal{D}_L Y^u \mathcal{U}_R \phi_- + \text{ h.c.} \\ &= \bar{\mathcal{D}}_L U^{d,\dagger} (U^d U^{u,\dagger}) Y^u_{diag} W^u \mathcal{U}_R \phi_- + \text{ h.c.}\end{aligned}$$

- In the charged currents flavour mixing occurs!
- Parametrized through the
  Cabbibo-Kobayashi-Maskawa Matrix:

$$V_{CKM} = U^d U^{u,\dagger}$$

Properties of the CKM Matrix

- V_{CKM} is unitary (by our construction)
- Number of parameters for n families
  - Unitary $n \times n$ matrix: n^2 real parameters
  - Freedom to rephase the $2n$ quark fields: $2n - 1$ relative phases
- $n^2 - 2n + 1 = (n-1)^2$ real parameters
  - * $(n-1)(n-2)/2$ are phases
  - * $n(n-1)/2$ are angles
- Phases are sources of CP violation
- $n = 2$: One angle, no phase $\to$ no CP violation
- $n = 3$: Three angles, one phase
- $n = 4$: Six angles, three phases

CKM Basics

- Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- Single phase δ: $U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}.$
- PDG CKM Parametrization:

$$V_{\mathrm{CKM}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12}$$

- Large Phases in $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$ and $V_{td} = |V_{td}| e^{i\beta}$

CKM Unitarity Relations

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Off diagonal zeros of $V_{CKM}^\dagger V_{CKM} = 1 = V_{CKM} V_{CKM}^\dagger$
- $V_{CKM}^\dagger V_{CKM} = 1 : \begin{cases} V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0 \\ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0 \\ V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^* = 0 \end{cases}$
- $V_{CKM} V_{CKM}^\dagger = 1 : \begin{cases} V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \\ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \\ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \end{cases}$

Wolfenstein Parametrization of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

- Expansion in $\lambda \approx 0.22$ up to λ^3
- A, ρ, η of order unity

Unitarity Triangle(s)

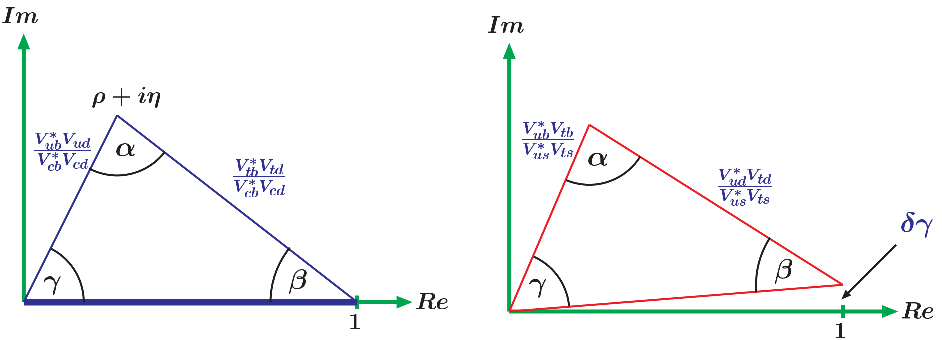
- The unitarity relations:
  Sum of three complex numbers = 0
- Triangles in the complex plane
- Only two out of the six unitarity relations involve terms of the same order in λ:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

- Both correspond to

$$A\lambda^3(\rho + i\eta - 1 + 1 - \rho - i\eta) = 0$$

- This is THE unitarity triangle ...



- Definition of the CKM angles α, β and γ
- To leading order Wolfenstein:

$$V_{ub} = |V_{ub}| e^{-i\gamma} \quad V_{tb} = |V_{tb}| e^{-i\beta}$$

all other CKM matrix elements are real.

- $\delta\gamma$ is order λ^5

- Aerea of the Triangle(s): Measure of CP Violation
- Invariant measure of CP violation:

$$\mathrm{Im}\Delta = \mathrm{Im}\, V_{ud} V_{td}^* V_{tb} V_{ub}^* = c_{12} s_{12} c_{13}^2 s_{13} s_{23} c_{23} \sin \delta_{13}$$

- Maximal possible value $\delta_{\max} = \frac{1}{6\sqrt{3}} \sim 0.1$
- CP Violation is a small effect:
  Measured value $\delta_{\exp} \sim 0.0001$
- CP Violation vanishes in case of degeneracies: (Jarlskog)

$$\begin{aligned} J &= \mathrm{Det}([M_u\,,\, M_d]) \\ &= 2i\mathrm{Im}\Delta (m_u - m_c)(m_u - m_t)(m_c - m_t) \\ &\quad \times (m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned}$$

Leptons in the Standard Model

- If the neutrinos are massless:
  - Only left handed neutrinos couple
  - Right handed neutrinos do not have any $SU(2)_L \times U(1)_Y$ quantum numbers
  - No mixing in the lepton sector
- Recent evidence for neutrino mixing:
  - Right handed components couple through the mass term
  - Mixing in the Lepton Sector
- It could be just a copy of the quark sector, but it may be different due to the properties of the right-handed neutrino

Multiplets and Quantum Numbers

- Left Handed Leptons: $SU(2)_L$ Doublets

$$L_1 = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \; L_2 = \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \; L_3 = \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

- Right Handed Leptons: $SU(2)_R$ Doublets

$$\ell_1 = \begin{pmatrix} \nu_{e,R} \\ e_R \end{pmatrix} \; \ell_2 = \begin{pmatrix} \nu_{\mu,R} \\ \mu_R \end{pmatrix} \; \ell_3 = \begin{pmatrix} \nu_{\tau,R} \\ \tau_R \end{pmatrix}$$

- Charge and Hypercharge

$$Y = T_{3,R} + \frac{1}{2}(B - L) = T_{3,R} - \frac{1}{2} \qquad q = T_{3,L} + Y$$

- Y (and q) project the lower component: Right handed Neutrinos: No charge, no Hypercharge

Majorana Fermions

- A “neutral” fermion can have a Majorana mass
- Charged fermions $\Leftrightarrow$ complex scalar fields
- Majorana fermion: “Real (= neutral) fermion”
- Definition of “complex conjugation” in this case: Charge Conjugation:

$$\psi \to \psi^c = C\bar{\psi}^T \quad C = i\gamma_2\gamma_0 = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}$$

- Properties of C

$$-C = C^{-1} = C^T = C^\dagger$$

- Majorana fermion: $\psi_{Majorana} = \psi^c_{Majorana}$
(Just as $\phi^* = \phi$ for a real scalar field)

Majorana Mass Terms

- Mass term for a Majorana fermion: The charge conjugate of a right handed fermion is left handed.
- Possible mass term

$$\mathcal{L}_{MM} = -\frac{1}{2} M \left(\bar{\nu}_R (\nu_R^c)_L + h.c. \right)$$

- Only for fields without $U(1)$ quantum numbers
- In the SM: only for the right handed neutrinos !
- Remarks:
  - The Majorana mass of the right handed neutrinos is NOT due to the Higgs mechanism.
  - Thus this majorana mass can be “large”
  - Natural explanation of the small neutrino masses: see-saw mechanism

See Saw Mechanism

- Simplification: One family: ν_L and ν_R
- Total Mass term: Dirac and Majorana mass

$$\begin{aligned}\mathcal{L}_{mass} &= -m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \\ &\quad -\frac{1}{2} M(\nu_R^T C \nu_R + \bar{\nu}_R C \bar{\nu}_R^T)\end{aligned}$$

- We use

$$\overline{(\nu_R^c)}_L (\nu_L^c)_R = \bar{\nu}_L \bar{\nu}_R$$

and the properties of the C matrix ...

$$\mathcal{L}_{mass} = -\frac{1}{2} \left(\bar{\nu}_L \; \overline{(\nu_R^c)}_L \right) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} (\nu_L^c)_R \\ \nu_R \end{pmatrix} + h.c.$$

- Diagonalization of the mass matrix:
  $\rightarrow$ Majorana mass eigenstates of the Neutrinos
  For $M \gg m$ we get

$$m_1 \approx \frac{m^2}{M} \quad m_2 \approx M$$

- One very heavy, practically right handed neutrino
- One very light, practically left handed neutrino
- At energies small compared to M:
  Majorana mass term for the left handed neutrino

$$\mathcal{L}_{mass} = -\frac{1}{2}\frac{m^2}{M}\left(\nu_L^T C \nu_L + \bar{\nu}_L C \bar{\nu}_L{}^T\right)$$

- Majorana mass is small if $M \gg m$

Right handed neutrinos in the Standard Model

- In case of three families: **Neutrino Mixing**
- Compact notation for the Leptons:

$$\mathcal{N}_{L/R} = \begin{bmatrix} \nu_{e,L/R} \\ \nu_{\mu,L/R} \\ \nu_{\tau,L/R} \end{bmatrix} \qquad \mathcal{E}_{L/R} = \begin{bmatrix} e_{L/R} \\ \mu_{L/R} \\ \tau_{L/R} \end{bmatrix}$$

- Dirac masses are generated by the Higgs mechanism: (as for the quarks)

$$\mathcal{L}_{DM}^{N} = -\mathcal{N}_L m^N \mathcal{N}_R + h.c.$$
$$\mathcal{L}_{DM}^{E} = -\mathcal{E}_L m^E \mathcal{E}_R + h.c.$$

- m^N: Dirac mass matrix for the neutrinos
- m^E: (Dirac) mass matrix for e, μ, τ

- Right handed neutrinos → Majorana mass term:

$$\mathcal{L}_{MM} = -\frac{1}{2}\left(N_R^T M C N_R + \bar{N}_R M C \bar{N}_R^T\right)$$

- M: (Symmetric) Majorana Mass Matrix
- This term is perfectly $SU(2)_L \otimes U(1)$ invariant
- Implementation of the see saw mechanism:
  Assume that all Eigenvalues of M are large
- Effective Theory at low energies:
  Only light, practically left handed neutrinos
- Effect of right handed neutrino:
  Majorana mass term for the light neutrinos

$$\mathcal{L}_{mass} = -\frac{1}{2}\left(N_L^T m^T M^{-1} m C N_L + \bar{N}_L m^T M^{-1} m C \bar{N}_L^T\right)$$

Lepton Mixing: PMNS Matrix

- Diagonalization of the Mass matrices:
  - Charged leptons:

$$m^E = U^\dagger m^E_{diag} W$$

  - Neutrinos: "Orthogonal" transformation:

$$m^T M^{-1} m = O^T m^\nu_{diag} O \text{ with } O^\dagger O = 1$$

- Again no Effect on neutral currents
- Charged Currents: Interaction with ϕ_+:

$$\frac{1}{v} \mathcal{N}_L m^E \mathcal{E}_R \phi_+ + \text{ h.c.}$$
$$= \frac{1}{v} \bar{\mathcal{N}}_L O^T (O^* U^\dagger) m^E_{diag} W \mathcal{E}_R \phi_+ + \text{ h.c.}$$

- A Mixing Matrix occurs:

$$V_{PMNS} = O^* U^\dagger$$

  Pontecorvo Maki Nakagawa Sakata Matrix
- V_{PMNS} is unitary like the CKM Matrix
- Left handed neutrinos are Majorana: No freedom to rephase these fields!
  - For n families: n^2 Parameters
  - Only n Relative phases free
  - $\longrightarrow n(n-1)$ Parameters
  - $n(n-1)/2$ are angles
  - $n(n-1)/2$ are phases: More sources for CP violation

- Almost like CKM: Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- A Dirac Phase δ and two Majorana Phases α_1 and α_2

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} \quad U_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{bmatrix}$$

- PMNS Parametrization: $V_{\rm PMNS} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12} U_\alpha$
- $\Theta_{23} \sim 45^\circ$ is “maximal” (atmospheric ν’s)
- $\Theta_{13} \sim 0$ is small (ν’s from reaktors)
- $\sin\Theta_{13} \sim 1/\sqrt{3}$ is large (solar ν’s)

Maltoni et al '04

parameter	best fit	2σ	3σ	5σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	≤ 0.035	≤ 0.054	≤ 0.11

$$V_{\text{PMNS}} \sim \begin{bmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \end{bmatrix} \sim \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{bmatrix}$$

- No Hierarchy !

Consequences of Lepton Mixing

- FCNC Processes in the leptonic Sector:

$$\tau \to \mu\gamma \quad \mu \to e\gamma \quad \tau \to eee \text{ etc.}$$
$$\nu_\tau \to \nu_e\gamma \quad \nu_\tau - \nu_e \text{ mixing}$$

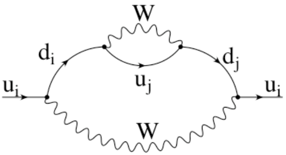
- Lepton Number Violation:
  Right handed Neutrinos are Majorana fermions:
  No conserved quantum number corresponding to the rephasing of the right handed neutrino fields
  Lepton number violation could feed via conserved $B - L$ into Baryon number violation
  Relation to the Baryon Asymmetry of the Universe ?

Peculiarities of SM Flavour Mixing

- Hierarchical structure of the CKM matrix
- Quark Mass spectrum ist widely spread
  $m_u \sim 10$ MeV to $m_t \sim 170$ GeV
- PMNS Matrix for lepton flavour mixing is not hierarchical
- Only the charged lepton masses are hierarchical
  $m_e \sim 0.5$ MeV to $m_\tau \sim 1772$ MeV
- Up-type leptons $\sim$ Neutrinos have very small masses
- (Enormous) Suppression of Flavour Changing Neutral Currents:
  $b \to s$, $c \to u$, $\tau \to \mu$, $\mu \to e$, $\nu_2 \to \nu_1$

Peculiarities of SM CP Violation

- Strong CP remains mysterious
- Flavour diagonal CP Violation is well hidden:
  e.g electric dipole moment of the neutron:
  At least three loops (Shabalin)



$$
\begin{aligned}
d_e &\sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \mathrm{Im}\Delta\, \mu^3 \\
&\sim 10^{-32} e\, cm \quad \text{with } \mu \sim 0.3\,\mathrm{GeV} \\
d_{\mathrm{exp}} &\leq 3.0 \times 10^{-26} e\, cm
\end{aligned}
$$

- Pattern of mixing and mixing induced CP violation determined by GIM: Tiny effects in the up quark sector
  - $\Delta C = 2$ is very small
  - Mixing with third generation is small: charm physics basically “two family”
  - $\rightarrow$ CP violation in charm is small in the SM
- Fully consistent with particle physics observations
- ... but inconsistent with matter-antimatter asymmetry

??? Many Open Questions ???

- Our Understanding of Flavour is unsatisfactory:
  - 22 (out of 27) free Parameters of the SM originate from the Yukawa Sector (including Lepton Mixing)
  - Why is the CKM Matrix hierarchical?
  - Why is CKM so different from the PMNS?
  - Why are the quark masses (except the top mass) so small compared with the electroweak VEV?
  - Why do we have three families?
- Why is CP Violation in Flavour-diagonal Processes not observed? (e.g. z.B. electric dipolmoments of electron and neutron)
- Where is the CP violation needed to explain the matter-antimatter asymmetry of the Universe?

Lecture 2
Theory Tools and Phenomenology

Thomas Mannel

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School on "Physics in the Standard Model and Beyond"

Tblisi, 28.09. - 30.09.2017

What is the Problem?

- Weak interaction: Transitions between quarks
- Observations: Transitions between Hadrons
- We have to deal with nonperturbative QCD
- Extraction of fundamental parameters (CKM Elements, CP Phases) requires precise predictions, including error estimates
- $\to$ Simple models are out ...
- Effective Field Theory methods
- QCD Sum Rules
- Lattice QCD

Contents

Effective Field Theories

- Weak decays:
  Very different mass scales are involved:
  - $\Lambda_{\mathrm{QCD}} \sim 200$ MeV: Scale of strong interactions
  - $m_c \sim 1.5$ GeV: Charm Quark Mass
  - $m_b \sim 4.5$ GeV: Bottom Quark Mass
  - $m_t \sim 175$ GeV and $M_W \sim 81$ GeV:
    Top Quark Mass and Weak Boson Mass
  - Λ_{NP} Scale of "new physics"
- At low scales the high mass particles / high energy degrees of freedom are irrelevant.
- Construct an "effective field theory" where the massive / energetic degrees of freedom are removed ("integrated out")

Integrating out heavy degrees of freedom

- ϕ: light fields, Φ: heavy fields with mass Λ
- Generating functional as a functional integral
  Integration over the heavy degrees of freedom

$$Z[j] = \int [d\phi][d\Phi] \exp\left(\int d^4x \, [\mathcal{L}(\phi, \Phi) + j\phi]\right)$$

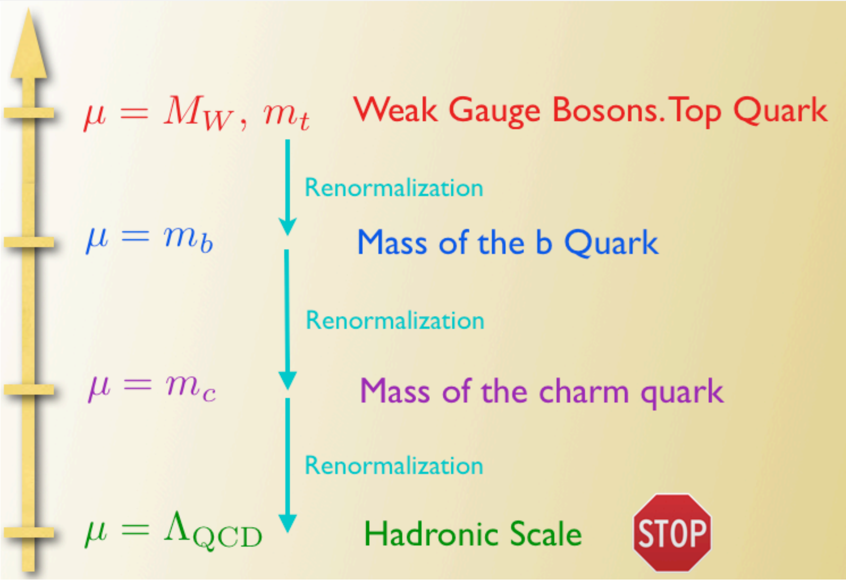
$$= \int [d\phi] \exp\left(\int d^4x \, [\mathcal{L}_{\text{eff}}(\phi) + j\phi]\right) \text{ with}$$

$$\exp\left(\int d^4x \, \mathcal{L}_{\text{eff}}(\phi)\right) = \int [d\Phi] \exp\left(\int d^4x \, \mathcal{L}(\phi, \Phi)\right)$$

- For length scales $x \gg 1/\Lambda$: local effective Lagrangian
- Technically: (Operator Product) Expansion in inverse powers of Λ

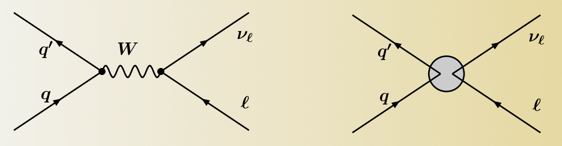
$$\mathcal{L}_{\rm eff}(\phi) = \mathcal{L}_{\rm eff}^{(4)}(\phi) + \frac{1}{\Lambda}\mathcal{L}_{\rm eff}^{(5)}(\phi) + \frac{1}{\Lambda^2}\mathcal{L}_{\rm eff}^{(6)}(\phi) + \cdots$$

- $\mathcal{L}_{\rm eff}$ is in general non-renormalizable, but ...
- $\mathcal{L}_{\rm eff}^{(4)}$ is the renormalizable piece
- For a fixed order in $1/\Lambda$: Only a finite number of insertions of $\mathcal{L}_{\rm eff}^{(4)}$ is needed!
- $\to$ can be renormalized
- Renormalizability is not an issue here


$\mu = M_W, m_t$
Weak Gauge Bosons. Top Quark
Renormalization
$\mu = m_b$
Mass of the b Quark
Renormalization
$\mu = m_c$
Mass of the charm quark
Renormalization
$\mu = \Lambda_{QCD}$
Hadronic Scale
STOP

Effective Weak Hamiltonian

- Start out from the Standard Model
- $W^\pm$, Z^0, top: much heavier than any hadron mass
- "integrate out" these particles at the scale $\mu \sim M_{\text{Hadron}}$



- W has zero range in this limit:

$$\langle 0 | T[W^*_\mu(x) W_\nu(y)] | 0 \rangle \to g_{\mu\nu} \frac{1}{M_W^2} \delta^4(x-y)$$

- Effective Interaction (Fermi Coupling)

$$H_{\text{eff}} = \frac{g^2}{\sqrt{2} M_W^2} V_{q'q} [\bar{q}' \gamma_\mu (1-\gamma_5) q][\bar{\nu}_\ell \gamma_\mu (1-\gamma_5) \ell] = \frac{4 G_F}{\sqrt{2}} V_{q'q} \, j_{\mu,\text{had}} \, j^\mu_{\text{lep}}$$

Decays of Hadrons

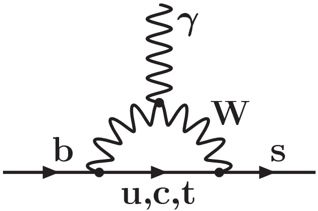
- Leptonic and semi-leptonic decays

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{q'q} [\bar{q}'\gamma_\mu \frac{(1-\gamma_5)}{2} q][\bar{\nu}_\ell \gamma_\mu \frac{(1-\gamma_5)}{2} \ell]$$

- Hadronic decays

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{q'q} V^*_{QQ'} [\bar{q}'\gamma_\mu \frac{(1-\gamma_5)}{2} q][\bar{Q}\gamma_\mu \frac{(1-\gamma_5)}{2} Q']$$

- Rare (FCNC) Decays: Loop Corrections (QCD and electroweak)



- Example: $b \to s\gamma$

$$\mathcal{A}(b \to s\gamma) = V_{ub}V_{us}^* f(m_u) + V_{cb}V_{cs}^* f(m_c) + V_{tb}V_{ts}^* f(m_t)$$

- In case of degenarate masses up-type masses:

$$\mathcal{A}(b \to s\gamma) = f(m)\left[V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^*\right] = 0$$

Renormalization Group Running

- $H_{\rm eff}$ is defined at the scale Λ, where we integrated out the particles with mass Λ: General Structure

$$H_{\rm eff} = \frac{4G_F}{\sqrt{2}} \lambda_{\rm CKM} \sum_k \hat{C}_k(\Lambda) \mathcal{O}_k(\Lambda)$$

- $\mathcal{O}_k(\Lambda)$: The matrix elements of $\mathcal{O}_k$ have to be evaluated ("normalized") at the scale Λ.
- $\hat{C}_k(\Lambda)$: Short distance contribution, contains the information about scales $\mu > \Lambda$
- Matrixelements of $\mathcal{O}_k(\Lambda)$: Long Distance Contribution, contains the information about scales $\mu < \Lambda$

- We could as well imagine a situation with a different definition of “long” and “short” distances, defined by a scale μ, in which case

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k C_k(\Lambda/\mu)\mathcal{O}_k(\mu)$$

- Key Observation: The matrix elements of H_{eff} are physical Quantities, thus cannot depend on the arbitrary choice of μ

$$0 = \mu \frac{d}{d\mu} H_{\text{eff}}$$

- compute this ....

$$0 = \sum_i \left(\mu \frac{d}{d\mu} C_i(\Lambda/\mu) \right) \mathcal{O}_i(\mu) + C_i(\Lambda/\mu) \left(\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) \right)$$

- Operator Mixing: Change in scale can turns the operator $\mathcal{O}_i$ into a linear combination of operators (of the same dimension)

$$\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) = \sum_j \gamma_{ij}(\mu) \mathcal{O}_j(\mu)$$

and so

$$\sum_i \sum_j \left(\left[\delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}(\mu) \right] C_i(\Lambda/\mu) \right) \mathcal{O}_j(\mu) = 0$$

- Assume: The operators $\mathcal{O}_j$ from a basis, then

$$\sum_i \left[\delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}^T(\mu) \right] C_j(\Lambda/\mu) = 0$$

- QCD: Coupling constant α_s depends on μ: β-function

$$\mu \frac{d}{d\mu} \alpha_s(\mu) = \beta(\alpha_s(\mu))$$

- C_j depend also on α_s

$$\mu \frac{d}{d\mu} = \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right)$$

- In an appropriate scheme γ_{ij} depend on μ only trough α_s:
  $\gamma_{ij}(\mu) = \gamma_{ij}(\alpha_s(\mu))$

- Renormalization Group Equation (RGE) for the coefficients

$$\sum_i \left[\delta_{ij}\left(\mu\frac{\partial}{\partial\mu}+\beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)+\gamma_{ij}^T(\alpha_s)\right] C_j(\Lambda/\mu,\alpha_s)=0$$

- This is a system of linear differential equations: $\rightarrow$ Once the initial conditions are known, the solution is in general unique
- RGE Running: Use the RGE to relate the coefficients at different scales

- The coefficients are at $\mu = \Lambda$ (at the "matching scale")

$$C_i(\Lambda/\mu = 1, \alpha_s) = \sum_n a_i^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^n \qquad \text{perturbative calculation}$$

- Perturbative calculation of the RG functions β and γ_{ij}

$$\beta(\alpha_s) = \alpha_s \sum_{n=0} \beta^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^{n+1} \qquad \gamma_{ij}(\alpha_s) = \sum_{n=0} \gamma_{ij}^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

- RG functions can be calculated from loop diagrams:

$$\beta^{(0)} = -\frac{2}{3}(33 - 2n_{\mathrm{f}}) \qquad \gamma_{ij} \text{ depends on the set of } \mathcal{O}_i$$

- Structure of the perturbative expansion of the coefficient at some other scale

$$
\begin{aligned}
&c_i(\Lambda/\mu, \alpha_s) = \\
&b_i^{00} \\
+\ & b_i^{11} \left(\frac{\alpha_s}{4\pi}\right) \ln\frac{\Lambda}{\mu} + b_i^{10} \left(\frac{\alpha_s}{4\pi}\right) \\
+\ & b_i^{22} \left(\frac{\alpha_s}{4\pi}\right)^2 \ln^2\frac{\Lambda}{\mu} + b_i^{21} \left(\frac{\alpha_s}{4\pi}\right)^2 \ln\frac{\Lambda}{\mu} + b_i^{20} \left(\frac{\alpha_s}{4\pi}\right)^2 \\
+\ & b_i^{33} \left(\frac{\alpha_s}{4\pi}\right)^3 \ln^3\frac{\Lambda}{\mu} + b_i^{32} \left(\frac{\alpha_s}{4\pi}\right)^3 \ln^2\frac{\Lambda}{\mu} + b_i^{31} \left(\frac{\alpha_s}{4\pi}\right)^3 \ln\frac{\Lambda}{\mu} + \cdots ,
\end{aligned}
$$

- LLA (Leading Log Approximation):
  Resummation of the b_i^{nn} terms

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} b_i^{nn} \left(\frac{\alpha_s}{4\pi}\right)^n \ln^n \frac{\Lambda}{\mu}$$

  $\to$ leading terms in the expansion of the RG functions
- NLLA (Next-to-leading log approximation):

$$C_i(\Lambda/\mu, \alpha_s) = \sum_{n=0}^{\infty} \left[b_i^{nn} + b_i^{n+1,n} \left(\frac{\alpha_s}{4\pi}\right)\right] \left(\frac{\alpha_s}{4\pi}\right)^n \ln^n \frac{\Lambda}{\mu}$$

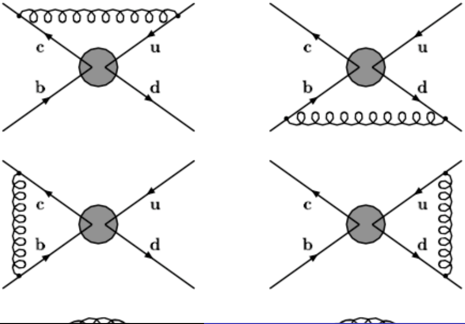
  $\to$ next-to-leading terms of the RG functions

- Typical Proceedure:
  - “Matching” at the scale $\mu = M_W$
  - “Running” to a scale of the order $\mu = m_b$
  - $\rightarrow$ includes operator mixing
- Resummation of the large logs $\ln(M_W^2/m_b^2)$
  - “Matching” at the scale $\mu = m_b$
  - “Running” to the scale m_c
- Resummation of the “large” logs $\ln(m_b^2/m_c^2)$
- ...
- Untli $\alpha_s(\mu)$ becomes too large ...

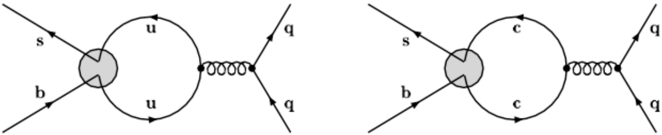
$H_{\rm eff}$ for b decays at low scales

- Effective interaction: $H_{\rm eff} = \frac{4G_F}{\sqrt{2}} \lambda_{\rm CKM} \sum_k \hat{C}_k(\Lambda)\mathcal{O}_k(\Lambda)$
- "Tree" Operators"

$$\mathcal{O}_1 = (\bar{c}_{L,i}\gamma_\mu s_{L,j})\,(\bar{d}_{L,j}\gamma_\mu u_{L,i}) \quad ,$$
$$\mathcal{O}_2 = (\bar{c}_{L,i}\gamma_\mu s_{L,i})\,(\bar{d}_{L,j}\gamma_\mu u_{L,j}) \quad .$$



- If two flavours are equal: QCD Penguin Operators



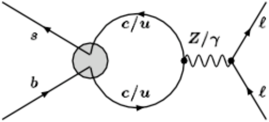
$$\mathcal{O}_3 = (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},i}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{L},j}\gamma^\mu q_{\mathrm{L},j}) \,,$$

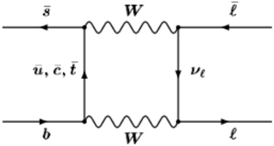
$$\mathcal{O}_4 = (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},j}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{L},j}\gamma^\mu q_{\mathrm{L},i}) \,,$$

$$\mathcal{O}_5 = (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},i}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{R},j}\gamma^\mu q_{\mathrm{R},j}) \,,$$

$$\mathcal{O}_6 = (\bar{s}_{\mathrm{L},i}\gamma_\mu b_{\mathrm{L},j}) \sum_{q=u,d,s,c,b} (\bar{q}_{\mathrm{R},j}\gamma^\mu q_{\mathrm{R},i}) \,.$$

- Electroweak Penguins:
  Replace the Gluon by a Z_0 or Photon: $\mathcal{P}_7 \cdots \mathcal{P}_{10}$
- Rare (FCNC) Processes:





$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}_{\mathrm{L},\alpha} \sigma_{\mu\nu} b_{\mathrm{R},\alpha}) F^{\mu\nu}$$

$$\mathcal{O}_8 = \frac{g}{16\pi^2} m_b (\bar{s}_{\mathrm{L},\alpha} T^a_{\alpha\beta} \sigma_{\mu\nu} b_{\mathrm{R},\alpha}) G^{a\mu\nu}$$

$$\mathcal{O}_9 = \frac{1}{2} (\bar{s}_{\mathrm{L}} \gamma_\mu b_{\mathrm{L}})(\bar{\ell} \gamma^\mu \ell)$$

$$\mathcal{O}_{10} = \frac{1}{2} (\bar{s}_{\mathrm{L}} \gamma_\mu b_{\mathrm{L}})(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Coefficients of the Operators (One Loop)

$C_i(\mu)$	$\mu = 10.0\,\mathrm{GeV}$	$\mu = 5.0\,\mathrm{GeV}$	$\mu = 2.5\,\mathrm{GeV}$
C_1	0.182	0.275	0.40
C_2	−1.074	−1.121	−1.193
C_3	−0.008	−0.013	−0.019
C_4	0.019	0.028	0.040
C_5	−0.006	−0.008	−0.011
C_6	0.022	0.035	0.055

$C_i(\mu)$	$\mu = 2.5\ \mathrm{GeV}$	$\mu = 5\ \mathrm{GeV}$	$\mu = 10\ \mathrm{GeV}$
$C_7^{\rm eff}$	−0.334	−0.299	−0.268
$C_8^{\rm eff}$	−0.157	−0.143	−0.131
$\frac{2\pi}{\alpha} C_9$	1.933	1.788	1.494

Heavy Quark Limit

Isgur, Wise, Voloshin, Shifman, Georgi, Grinstein, ...

- $1/m_Q$ Expansion: Substantial Theoretical Progress!
- Static Limit: $m_b,\ m_c \to \infty$ with fixed (four)velocity

$$v_Q = \frac{p_Q}{m_Q}, \qquad Q = b, c$$

- In this limit we have

$$\left.\begin{array}{r} m_{Hadron} = m_Q \\ p_{Hadron} = p_Q \end{array}\right\} v_{Hadron} = v_Q$$

- For $m_Q \to \infty$ the heavy quark does not feel any recoil from the light quarks and gluons (Cannon Ball)
- This is like the H-atom in Quantum Mechanics !!

Heavy Quark Symmetries

- The interaction of gluons is identical for all quarks
- Flavour enters QCD only through the mass terms
  - $m \to 0$: (Chiral) Flavour Symmetry (Isospin)
  - $m \to \infty$ Heavy Flavour Symmetry
  - Consider b and c heavy: Heavy Flavour SU(2)
- Coupling of the heavy quark spin to gluons:

$$H_{int} = \frac{g}{2m_Q} \bar{Q}(\vec{\sigma} \cdot \vec{B})Q \quad \stackrel{m_Q \to \infty}{\longrightarrow} \quad 0$$

  - Spin Rotations become a symmetry
  - Heavy Quark Spin Symmetry: SU(2) Rotations
- Spin Flavour Symmetry Multiplets

Mesonic Ground States

Bottom:

$$|(b\bar{u})_{J=0}\rangle = |B^-\rangle \qquad |(b\bar{u})_{J=1}\rangle = |B^{*-}\rangle$$

$$|(b\bar{d})_{J=0}\rangle = |\overline{B}^0\rangle \qquad |(b\bar{d})_{J=1}\rangle = |\overline{B}^{*0}\rangle$$

$$|(b\bar{s})_{J=0}\rangle = |\overline{B}_s\rangle \qquad |(b\bar{s})_{J=1}\rangle = |\overline{B}_s^*\rangle$$

Charm:

$$|(c\bar{u})_{J=0}\rangle = |D^0\rangle \qquad |(c\bar{u})_{J=1}\rangle = |D^{*0}\rangle$$

$$|(c\bar{d})_{J=0}\rangle = |D^+\rangle \qquad |(c\bar{d})_{J=1}\rangle = |D^{*+}\rangle$$

$$|(c\bar{s})_{J=0}\rangle = |D_s\rangle \qquad |(c\bar{s})_{J=1}\rangle = |D_s^*\rangle$$

Baryonic Ground States

$$\left|[(ud)_0 Q]_{1/2}\right\rangle = |\Lambda_Q\rangle$$

$$\left|[(uu)_1 Q]_{1/2}\right\rangle, \left|[(ud)_1 Q]_{1/2}\right\rangle, \left|[(dd)_1 Q]_{1/2}\right\rangle = |\Sigma_Q\rangle$$

$$\left|[(uu)_1 Q]_{3/2}\right\rangle, \left|[(ud)_1 Q]_{3/2}\right\rangle, \left|[(dd)_1 Q]_{3/2}\right\rangle = |\Sigma_Q^*\rangle$$

$$\left|[(us)_0 Q]_{1/2}\right\rangle, \left|[(ds)_0 Q]_{1/2}\right\rangle = |\Xi_Q\rangle$$

$$\left|[(us)_1 Q]_{1/2}\right\rangle, \left|[(ds)_1 Q]_{1/2}\right\rangle = |\Xi'_Q\rangle$$

$$\left|[(us)_1 Q]_{3/2}\right\rangle, \left|[(ds)_1 Q]_{3/2}\right\rangle = |\Xi_Q^*\rangle$$

$$\left|[(ss)_1 Q]_{1/2}\right\rangle = |\Omega_Q\rangle \qquad \left|[(ss)_1 Q]_{3/2}\right\rangle = |\Omega_Q^*\rangle$$

Wigner Eckart Theorem for HQS

- HQS imply a “Wigner Eckart Theorem”

$$\left\langle H^{(*)}(v)\right| Q_v \Gamma Q_{v'} \left|H^{(*)}(v')\right\rangle = C_\Gamma(v, v')\xi(v \cdot v')$$

with $H^{(*)}(v) = D^{(*)}(v)$ or $B^{(*)}(v)$

- $C_\Gamma(v, v')$: Computable Clebsh Gordan Coefficient
- $\xi(v \cdot v')$: Reduced Matrix Element
- $\xi(v \cdot v')$: universal non-perturbative Form Faktor: Isgur Wise Funktion
- Normalization of ξ at $v = v'$:

$$\xi(v \cdot v' = 1) = 1$$

Heavy Quark Effective Theory

- The heavy mass limit can be formulated as an effective field theory
- Expansion in inverse powers of m_Q
- Define the static field h_v for the velocity v

$$h_v(x) = e^{im_Q v\cdot x}\frac{1}{2}(1+\not{v})b(x) \qquad p_Q = m_Q v + k$$

- HQET Lagrangian

$$\mathcal{L} = \bar{h}_v(iv\cdot D)h_v + \frac{1}{2m_Q}\bar{h}_v(i\not{D})^2 h_v + \cdots$$

- Dim-4 Term: Feynman rules, loops, renormalization...

Application: Determination of V_{cb} from $B \to D^{(*)}\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^* \ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega)(\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D \ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\text{QED}}\eta_A\left[1 + \delta_{1/\mu^2} + \cdots\right](\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$
$$\mathcal{G}(1) = \eta_{\text{QED}}\eta_V\left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$

$B \to D^{(*)}$ Form Factors

- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.903 \pm 0.13}$$

$$\boxed{\mathcal{G}(1) = 1.033 \pm 0.018 \pm 0.0095}$$

A. Kronfeld et al.

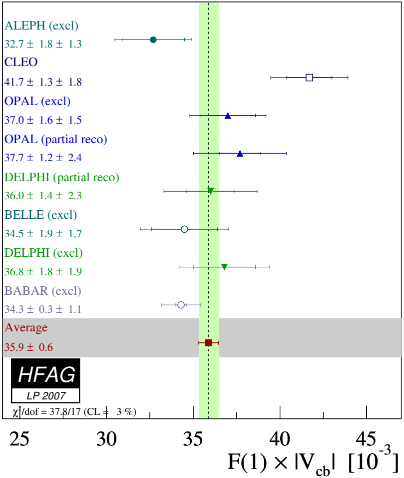
- Zero Recoil Sum Rules

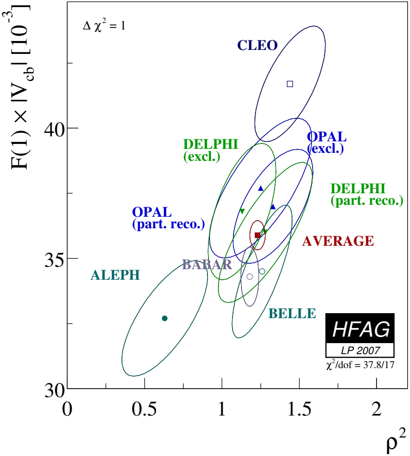
$$\boxed{\mathcal{F}(1) = 0.86 \pm 0.04}$$

$$\boxed{\mathcal{G}(1) = 1.04 \pm 0.02}$$

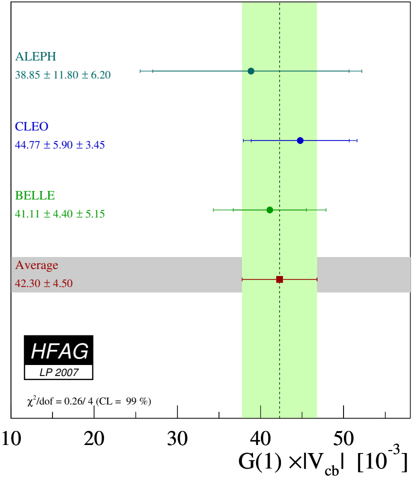
P. Gambino et al.

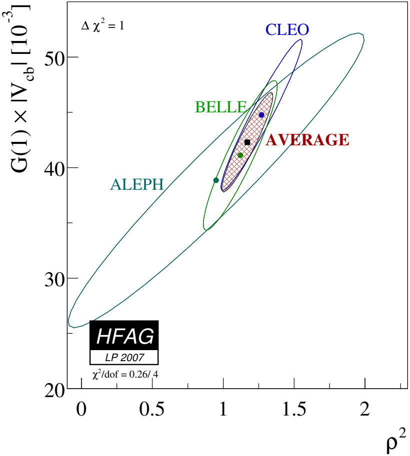
$B \to D^* \ell \bar{\nu}_\ell$





$B \to D \ell \bar{\nu}_\ell$





Inclusive Decays: Heavy Quark Expansion

Operator Product Expansion = Heavy Quark Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v) \rangle|^2$$

$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, \langle B(v) | T\{\mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0)\} | B(v) \rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, e^{-im_b v \cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}^\dagger_{eff}(0)\} | B(v) \rangle$$

- Last step: $p_b = m_b v + k$,
  Expansion in the residual momentum k

- Perform an OPE: m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\}$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\to$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\to$ Perturbation theory!

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Heavy Quark Symmetries
- Γ_2 is expressed in terms of two parameters

$$2M_H\mu_\pi^2 = -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle$$
$$2M_H\mu_G^2 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$2M_H\rho_D^3 = -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle$$
$$2M_H\rho_{LS}^3 = \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle$$

ρ_D: Darwin Term and ρ_{LS}: Chromomagnetic moment

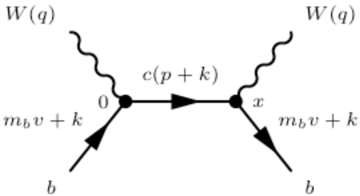
New: $1/m_b^4$ Contribution Γ_4 (Dassinger, Turczyk, M.)

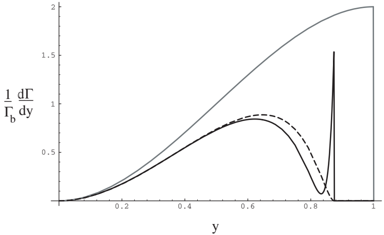
- Five new parameters:

$$\langle \vec{E}^2 \rangle : \quad \text{Chromoelectric Field squared}$$
$$\langle \vec{B}^2 \rangle : \quad \text{Chromomagnetic Field squared}$$
$$\langle (\vec{p}^2)^2 \rangle : \quad \text{Fourth power of the residual } b \text{ quark momentum}$$
$$\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle : \quad \text{Mixed Chromomag. Mom. and res. Mom. sq.}$$
$$\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle : \quad \text{Mixed Chromomag. field and res. helicity}$$

- Some of these can be estimated in naive factorization

Spectra of Inclusive Decays





- Endpoint region: $\rho = m_c^2/m_b^2$, $y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \Theta(1-y-\rho)\left[2 + \frac{\lambda_1}{(m_b(1-y))^2}\left(\frac{\rho}{1-y}\right)^2\left\{3 - 4\frac{\rho}{1-y}\right\}\right]$$

- Reliable calculation in HQE possible for the moments of the spectrum

Application: V_{cb} from $b \to c\ell\bar{\nu}$ inclusive

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- QCD insprired modelling for the HQE matrix elements
- New: Complete α_s/m_b^2, including the μ_G terms
  Alberti, Gambino, Nandi (arXiv:1311.7381)
  ThM, Pivovarov, Rosenthal (arXiv:1405.5072, arXiv:1506.08167)
- This was the remaining parametrically largest uncertainty

- Alberti et al.: **Phys.Rev.Lett. 114 (2015) 6, 061802** and **JHEP 1401 (2014) 147**
    - Calculation of the differential rate including the charm mass
    - partially numerical calculation
- ThM, Pivovarov, Rosenthal: **Phys.Lett. B741 (2015) 290-294**
    - Fully analytic calculation
    - limit $m_c \to 0$
    - Possibility to include m_c in a Taylor series
- Results do agree, surprisingly steep m_c dependence

Result for V_{cb}

- Inclusive Decay (HQE / OPE)

$$\boxed{V_{cb} = 42.21 \pm 0.78}$$ (Gambino et al. 2015)

- Exclusive decay (Lattice FF)

$$\boxed{V_{cb} = 39.36 \pm 0.78}$$ (Fermilab/Milc 2015)

- Exclusive decay (Zero Recoil Sum rule)

$$\boxed{V_{cb} = 41.4 \pm 0.9}$$ (Gambino et al. 2015)

Soft Collinear Effective Theory

- Problem: How to deal with “energetic” light degrees of freedom = Endpoint regions of the spectra ?
- More than two scales involved!
- Inclusive Rates in the Endpoint become (Korchemski, Sterman)

$$d\Gamma = H * J * S$$

with * = Convolution

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$

Basics of Soft Collinear Effective Theory

- Heavy-to-light decays:
  Kinematic Situations with energetic light quarks
  hadronizing into jets or energetic light mesons
  p_{fin}: Momentum of a light final state meson

$$p_{\rm fin}^2 \sim \mathcal{O}(\Lambda_{\rm QCD} m_b) \quad v \cdot p_{\rm fin} \sim \mathcal{O}(m_b)$$

- Use light-cone vectors $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$:

$$p_{\rm fin} = \frac{1}{2}(n \cdot p_{\rm fin})\bar{n} \quad \text{and} \quad v = \frac{1}{2}(n + \bar{n})$$

- Momentum of a light quark in such a meson:

$$p_{\rm light} = \frac{1}{2}[(n \cdot p_{\rm light})\bar{n} + (\bar{n} \cdot p_{\rm light})n] + p_{\rm light}^{\perp}$$

SCET Power Counting

- Define the parameter $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$
- The light quark invariant mass (or virtuality) is assumed to be

$$p_{\rm light}^2 = (n \cdot p_{\rm light})(\bar{n} \cdot p_{\rm light}) + (p_{\rm light}^\perp)^2 \sim \lambda^2 m_b^2$$

- The components of the quark momentum have to scale as

$$(n \cdot p_{\rm light}) \sim m_b \quad (\bar{n} \cdot p_{\rm light}) \sim \lambda^2 m_b \qquad p_{\rm light}^\perp \sim \lambda m_b$$

A brief look at SCET (Bauer, Stewart, Pirjol, Beneke, Feldmann ...)

- QCD quark field q is split into a collinear component ξ and a soft one with $\xi = \frac{1}{4} \not{n}_- \not{n}_+ q$
- The Lagrangian $\mathcal{L}_{\rm QCD} = \bar{q}(i\not{D})q$ is rewritten in terms of the collinear field

$$\mathcal{L} = \frac{1}{2}\bar{\xi}\not{n}_+(in_- D)\xi - \bar{\xi} i\not{D}_\perp \frac{1}{in_+ D + i\epsilon} \frac{\not{n}_+}{2} i\not{D}_\perp \xi$$

- Expansion according to the above power couning:

$$in_+ D = in_+ \partial + gn_+ A_{\rm c} + gn_+ A_{\rm us} = in_+ D_{\rm c} + gn_+ A_{\rm us}$$

- Leading $\mathcal{L}$ becomes non-local: Wilson lines

Practical Consequences of SCET

- Similar to HQS: Relations between for factors at large momentum transfer

$$\langle B(v)|\bar{b}\Gamma q|\pi(p)\rangle \propto \zeta(vp),\ \zeta_{//}(vp),\ \zeta_{\perp}(vp)$$

  For energetic pion only three independent form factors (Charles et al.)

- Correction can be calculated as in HQET

Basic Idea (Shifman, Vainshtein, Zakharaov, 1978)

- Start from a suitably chosen correlation function, e.g.

$$T(q^2) = \int d^4x\, e^{-iqx} \langle 0 | T[j(x) j^\dagger(0) | 0 \rangle$$

- This can be calculated perturbatively as $q^2 \to -\infty$.
- On the other hand, it has a dispersion relation

$$T(q^2) = \int \frac{ds}{2\pi} \frac{\rho(s)}{s - q^2 + i\epsilon} + \text{possible subtractions}$$

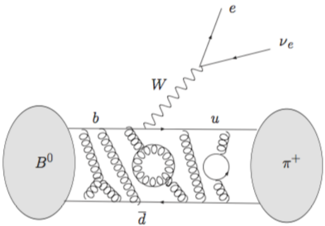
with $\rho(s) \sim \langle 0 | j(x) j^\dagger(0) 0 \rangle = \sum_n \langle 0 | j(x) | n \rangle \langle n | j^\dagger(0) 0 \rangle$

- Estimates for $\langle 0 | j(x) | n \rangle$ from e.g. positivity statements

Application: Determination of V_{ub} from exclusive $b \to u\ell\bar{\nu}$

- $B \to \pi\ell\nu_\ell$, determination of $|V_{ub}|$

  - decay amplitude parametrized by hadronic form factors



$$\langle \pi^+(p)|\bar{u}\gamma_\mu b|\bar{B}^0(p+q)\rangle = f^+_{B\pi}(q^2)\left[...\right]_\mu + f^0_{B\pi}(q^2)\left[...\right]_\mu$$

  - $|V_{ub}|$ determination [BaBar,Belle]

$$\left(\frac{1}{\tau_B}\right)\frac{dBR(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

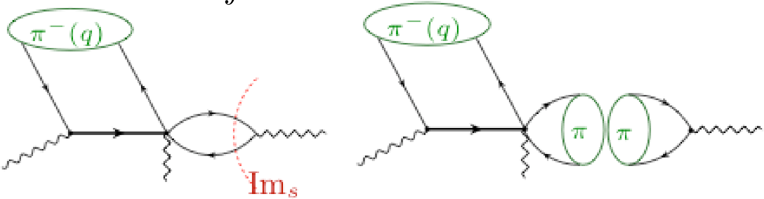
$0 < q^2 < (m_B - m_\pi)^2 \sim 26 \text{ GeV}^2$,

  - form factors accessible in lattice QCD at $q^2 \gtrsim 16 \text{ GeV}^2$

$f_+(q^2)$ from QCD Sum Rules (Ball, Zwicky, Khodjamirian, ...)

- Dispersion Relation and Light Cone Expansion
- Study a Correlation Function

$$F_\lambda(p,q) = i \int d^4x \; e^{ipx} \langle \pi^+(q) | T\{\bar{u}\gamma_\lambda b(x) \; m_b \bar{b} i \gamma_5 d(0)\} | 0 \rangle$$



- Yields an estimate for $f_B f_+(q^2)$
- Limited to small q^2

Results from LCSR

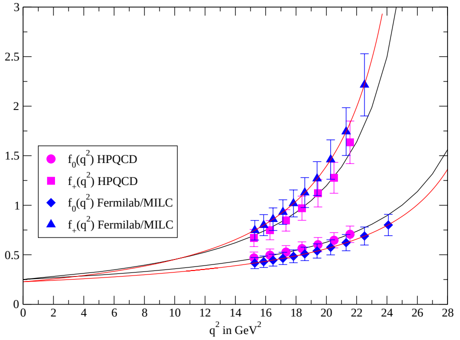
- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times$$
$$\left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Lattice QCD for Heavy to Light Form Factors

- Results reliable for large q^2
- Unquenched results are available
- Extrapolation to small q^2 by a pole model Becirevic, Kaidalov



Rate for $q^2 \geq 16\,\mathrm{GeV}^2$

$$|V_{ub}|^2 \times (1.31 \pm 0.33)\,\mathrm{ps}^{-1}$$
$$|V_{ub}|^2 \times (1.80 \pm 0.48)\,\mathrm{ps}^{-1}$$

(HPQCD / Fermilab MILC)

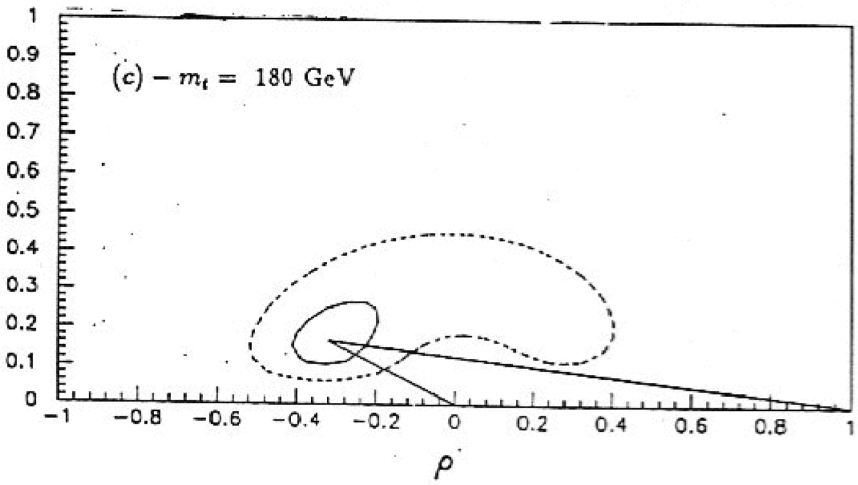
Status of V_{ub}

- From QCD LCSR: $V_{ub} = (3.32 \pm 0.26) \times 10^{-3}$
- PDG 2104:
  - Inclusive (LC-OPE): $V_{ub} = (4.41 \pm 0.25) \times 10^{-3}$
  - Exclusive (Combined): $V_{ub} = (3.28 \pm 0.29) \times 10^{-3}$
- This is the famous tension between the $V_{ub}s$

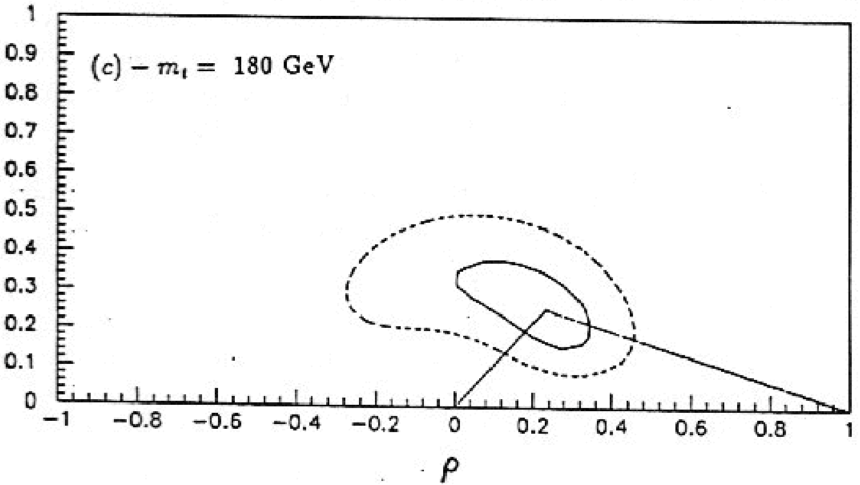
The history of the UT since $\sim$ 1993

- Situation in 1993:
  - HQET was still young ($\sim$ 3 years)
  - Hadronic Matrix elements for $\Delta m_d \sim f_B^2$ were quite uncertain
  - V_{ub}/V_{cb} was known at the level of $\sim$ 20%
  - The top quark mass was still $m_t \sim (140 \pm 40)$ GeV
  - No CP violation has been observed except ϵ_K
- The UT still could have been "flat"

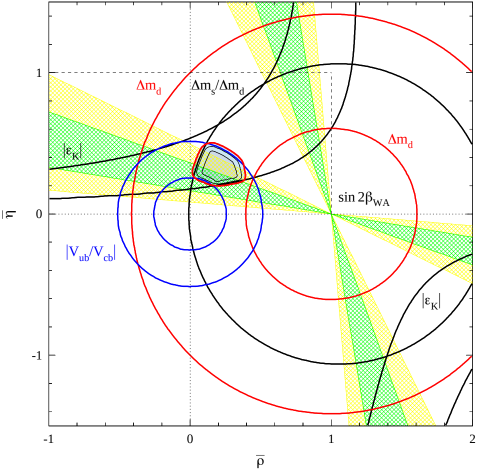
Unitarity triangle 1993: $f_B = 135 \pm 25$ MeV

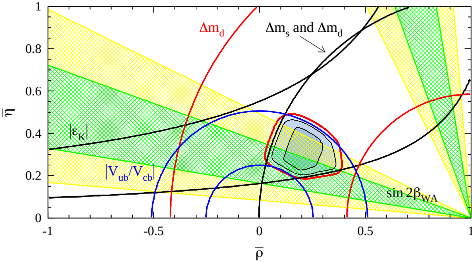


Unitarity triangle 1993: $f_B = 200 \pm 30$ MeV

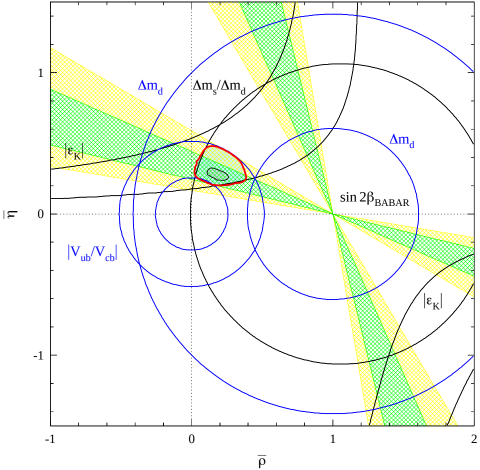


2001: First observation of “Non-Kaon CPV”

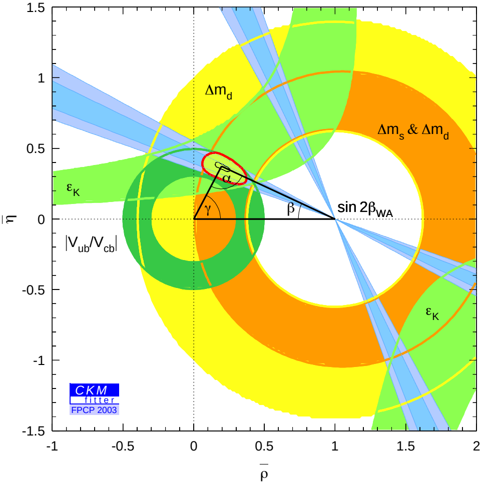




Unitarity Triangle 2001

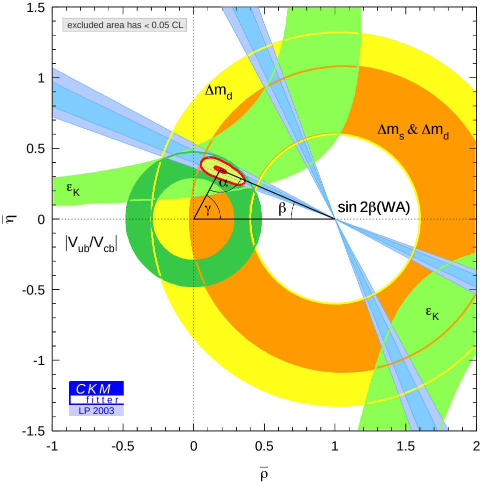


Unitarity Triangle 2002



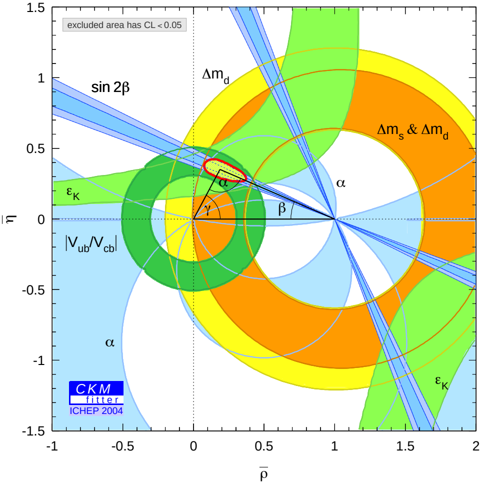
- Some improvement of V_{ub}/V_{cb} through the Heavy Quark Expansion
- More data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$

Unitarity Triangle 2003



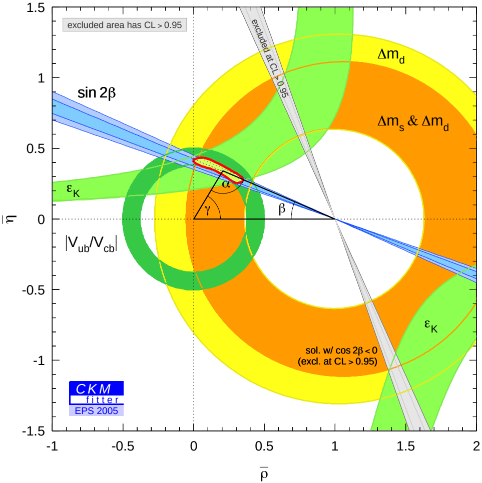
- Slight improvement of $f_B^2 B_B$ from lattice calculations
- Still more data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$
- Central value of V_{ub}/V_{cb} slightly moved

Unitarity Triangle 2004



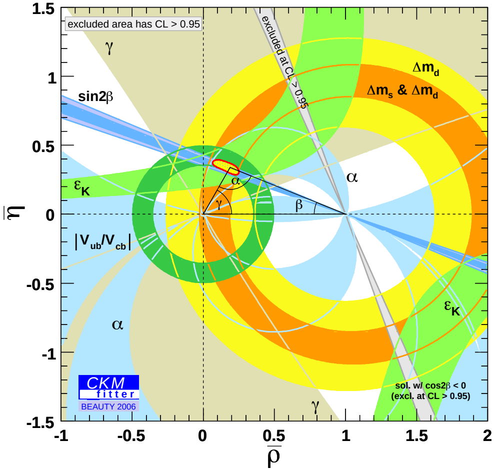
- More improvement of $f_B^2 B_B$ from lattice calculations
- Still more data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$
- First constraints on the angle α from $B \to \rho\rho$

Unitarity Triangle 2005



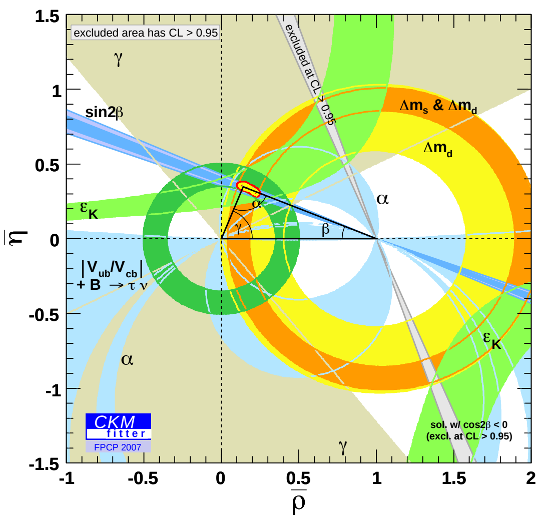
- Still more data on $\mathcal{A}_{\rm CP}(B \to J/\Psi K_s)$
- Exclusion of the “wrong branch” of β
- Dramatic Improvement of V_{ub} from the HQE

Unitarity Triangle 2006

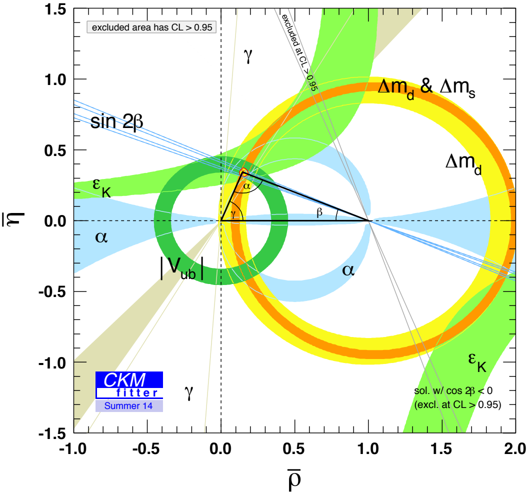


- TEVATRON measurement of Δm_s
- Tighter constraints on α
- First constraints on γ from CPV in $B \to K\pi$

Unitarity traingle 2007



Unitarity Triangle Now



Conclusions on Tools and Phenomenoogy

- There are more phenomenological methods which have not been mentioned
  - QCD factorization for non-leptonic decays
  - HQE calculations of neutral meson mixings
  - Multibody decays
  - CP Violation measurements and phenomenology
- We are in the era of precision flavor physics
- Sensitivity to NEW PHYSICS?

Lecture 3
Flavor Beyond the Standard Model

Thomas Mannel

Theoretische Physik I   Universität Siegen

School on "Physics in the Standard Model and Beyond"

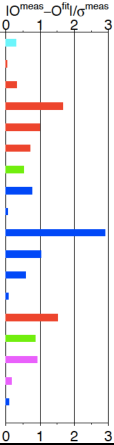
Tblisi, 28.09. - 30.09.2017

Contents

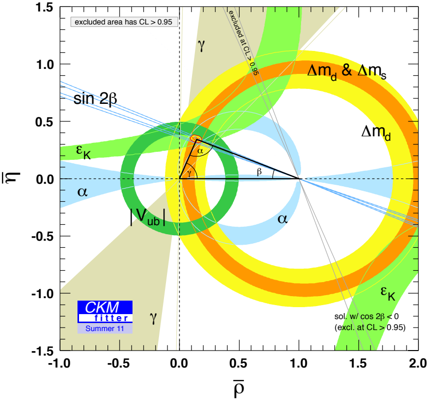
Why Study Flavour Physics?

- The Standard Model passed all tests
  up to the 100 GeV Scale:
- LEP: test of the gauge Structure
- Flavour factories: test of the Flavour Sector

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01646
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1482
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1039
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0743
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1482
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.023	80.378
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.27

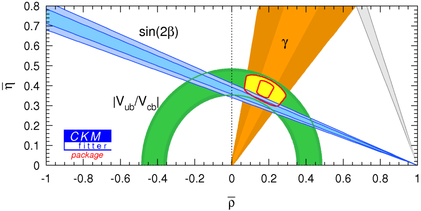


July 2011

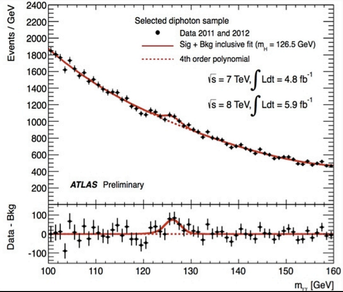


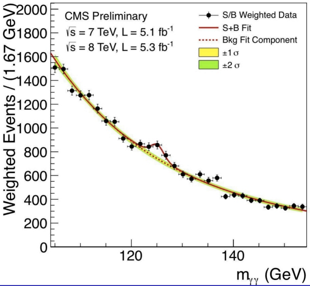
No significant deviation has been found (yet)!

... only a few "tensions"
(= Observables off by 2σ
or even less)



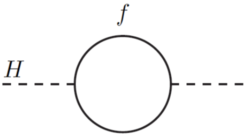
LHC will perform a direct test of the TeV Scale





Why do we believe in TeV Physics?

- Theoretical argument:
- Stabilization of the electroweak scale:

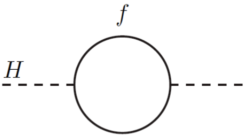


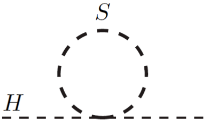
- Quadratic Dependence on the cut-off

$$\Delta m_H^2 = -\frac{\lambda_f^2}{8\pi^2}\Lambda_{\mathrm{UV}}^2$$

- Drives the Higgs mass up to the UV cut off $\Lambda_{\mathrm{UV}} \sim M_{\mathrm{PL}}$

- Stabilization at the TeV scale: e.g. through SUSY:





- Only logarithmic divergence

$$\Delta m_H^2 = m_{\rm soft}^2 \frac{\lambda}{16\pi^2} \ln\left(\frac{\Lambda_{\rm UV}}{m_{\rm soft}}\right)$$

- $m_{\rm soft} \sim \mathcal{O}(\text{TeV})$:
  Splitting between particles and particles

- How strong are these arguments?
- Could there something be wrong with our understanding of
  - electroweak symmetry breaking?
  - scale and conformal invariance? (c.f. Lee Wick Model)
  - ...
- Does flavour tell us something about this? .... and what?

What can Flavour tell us?

- Effective field theory picture:
- Standard model (without right handed ν's) is the (dim-4) starting point.
- Any new physics manifests itself as higher dimensional operators:

$$\mathcal{L} = \mathcal{L}^{SM}_{\dim 4} + \mathcal{L}_{\dim 5} + \mathcal{L}_{\dim 6} + \cdots$$

- $\mathcal{L}_{\dim \mathrm{n}}$ are suppressed by large mass scales

$$\mathcal{L}_{\dim \mathrm{n}} = \frac{1}{\Lambda^{n-4}} \sum_i C_n^{(i)} O_n^{(i)}$$

$O_n^{(i)}$: Operators of dimension n,
$SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariant
$C_n^{(i)}$: dimensionless couplings

Quark Flavour Physics

- For Quarks there is no contribution to $\mathcal{L}_{\mathrm{dim}\,5}$
- Some of the $O_j^{(n)}$ mediate $\Delta F = 2$ flavour transitions:

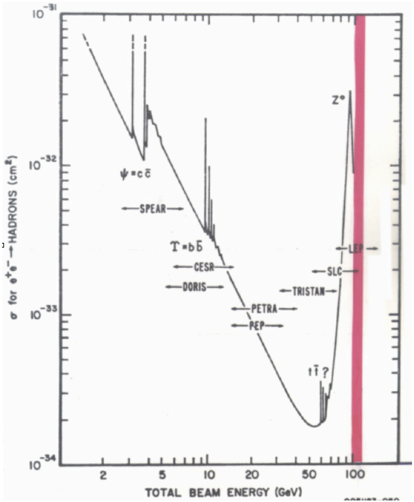
$$
\begin{aligned}
O_1^{(6)} &= (\bar{s}_L \gamma_\mu d)(\bar{s}_L \gamma^\mu d) && \text{(Kaon Mixing)} \\
O_2^{(6)} &= (\bar{b}_L \gamma_\mu d)(\bar{b}_L \gamma^\mu d) && (B_d \text{ Mixing}) \\
O_3^{(6)} &= (\bar{b}_L \gamma_\mu 2)(\bar{b}_L \gamma^\mu s) && (B_s \text{ Mixing}) \\
O_4^{(6)} &= (\bar{c}_L \gamma_\mu u)(\bar{c}_L \gamma^\mu u) && (D \text{ Mixing})
\end{aligned}
$$

- $\Lambda \sim 1000$ TeV from Kaon mixing ($C_i = 1$)
- $\Lambda \sim 1000$ TeV from D mixing
- $\Lambda \sim 400$ TeV from B_d mixing
- $\Lambda \sim 70$ TeV from B_s mixing

- “New physics” is around the corner??
- Are the flavour data a hint at a new physics scale well above the TeV scale?
- ... there are a few corners where $\mathcal{O}(1)$ flavour effects are still possible, c.f. Charm CPV
- Are there lessons from history?

The Top Quark Story

- First indirect hint to a heavy top quark: $B - \overline{B}$ Oscillation of ARGUS (1987)
- The world in 1987 ("PETRA Days"): The top was believed to be at $\sim$ 25 GeV
  ... based on good theoretical arguments
- ARGUS could not have seen anything with a 25 GeV Top (within SM)



- The consequences:
  - (–) No Toponium
  - (–) No Top quark discovery at LEP and SLC
  - (–) No “New Physcis $\mathcal{O}$(30 GeV)” just around the corner
  - (+) CP violation in the *B* sector may become observable
  - (+) GIM is weak for bottom quarks
- This was actually good for Flavour Physics ...
- GIM suppressed decays as a probe for large scales
- From current data: TeV “New Physics” must have a flavour structure close to the one of the SM
- $\rightarrow$ Concept of “Minimal Flavour Violation” (MFV)

Hints from the leptonic sector

- $\mathcal{L}^{SM}_{\dim 4}$ does not have a right handed neutrino
- ... thus no mixing for the leptons
- Discovery of Neutrino Oscillations:
  Nontrivial Flavour Physics of Leptons
- Important observation: The combination

$$N_i = (H^{c,\dagger} L_i)\,, \quad L_i = \begin{pmatrix} \nu_{L,i} \\ \ell_{L,i} \end{pmatrix}, \; H^c = (i\tau^2) H^*\,, H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

has no SM Quantum numbers

- This allows for a Unique dim -5 Operator:
  Generates Majorana masses for the ν's

$$\mathcal{L}_{\rm dim\,5} = \frac{1}{\Lambda_{\rm LNV}} \sum_{ij} C_5^{ij} (\bar{L}_j H^c)^c (H^{c,\dagger} L_i)$$

- Generates a mixing matrix for the leptons (PMNS Matrix), analogous to the CKM Matrix
- This term is Lepton Number Violating, related to the scale $\Lambda_{\rm LNV}$
- Small Neutrino masses: $\Lambda_{\rm LNV}$ must be high, almost as big as the GUT scale?
- Hopefully $\Lambda_{\rm QFV}$ and $\Lambda_{\rm LFV}$ is not that high!

Minimal Flavour Violation

- Flavour Violation of TeV "new physics" must be very close to one of the Standard Model
- Concept of "minimal flavour violation":
  All Flavour Violation (and CP violation) is CKM like
  (D'Ambrosio et al. '02, Ciuchini et al. '98, Buras et al. '00)
- More precise definition
  D'Ambrosio et al., hep-ph/0207036
- Leptonic Sector has also been considered as well
  Grinstein et al., hep-ph/0507001, hep-ph/0601111
- Standard Model is Minimally Flavour Violating per definition

- Most of the commonly used new physics models are constructed to solve any others but the flavor problems!
- .. but we hope to see something at LHC!
- So it has to be MFV!

Flavour Symmetry: Quarks

- Largest Quark Flavour Symmetry commuting with the Gauge Group of the Standard Model

$$G_F = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

with

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim (3,1,1) \quad U_R \sim (1,3,1) \quad D_R \sim (1,1,3)$$

- G_F is *explicitely* broken by the Yukawa couplings

$$\mathcal{L}_{\text{Yuk}} = \bar{Q}_L H Y_D D_R + \bar{Q}_L \widetilde{H} Y_U U_R$$

- Diagonalization of the Yukawa Couplings

$$Y_D^{\text{diag}} = V_{DL}^\dagger Y_D V_{DR} \quad Y_U^{\text{diag}} = V_{UL}^\dagger Y_U V_{UR}$$

- Leads after Spontaneous Symmetry Breaking to diagonal Mass Matrices for the Quarks
- Note that $V_{UR} \in SU(3)_{U_R}$ and $V_{DR} \in SU(3)_{DR}$
- ... but both V_{UL} and V_{DL} should be $\in SU(3)_{Q_L}$
- this leads to a relative and observable mismatch and

$$V_{\text{CKM}} = V_{U_L}^\dagger V_{D_L}$$

- Using mass eigenstates, V_{CKM} appears as the matrix of charged current couplings.

Spurions

- Trick to parametrize explicit symmetry breaking: Introduce “Spurions”
- Spurion: Field with a well defined transformation under the symmetry to be explicitely broken.
- Write all terms that are allowed by the symmetry with a finite number of insertions of the spurion field(s)
- “Freeze” the spurion field(s) to a nonzero value: “vacuum expectation value”
- Explicit Symmetry breaking = Spontaneous Symmetry Breaking without the Higgs degrees of freedom
- Small symmetry breaking: Power counting for the spurion insertions is needed.

Yukawa Couplings as Spurions

- Interpret the Yukawa couplings as spurion fields transforming as

$$Y_U \sim (3, \bar{3}, 1) \quad Y_U \sim (3, 1, \bar{3})$$

- In this way the Yukawa terms become formally invariant under G_F
- "Freezing" the Yukawa couplings to the observed values breaks G_F explicitely.
- Minimal Flavour Violation: The two spurions Y_U and Y_D are the only sources of flavour violation.

Example $B \to X_s\gamma$ in MFV

- The $b \to s\gamma$ decay is a $D_R \to D_L$ transition.
- $\bar{Q}_L D_R$ is not invariant under G_F
- $\bar{Q}_L Y_D D_R \to \bar{D}_L m_d^{\mathrm{diag}} D_R$ is flavour diagonal.
- $\bar{Q}_L Y_U Y_U^\dagger Y_D D_R \to \bar{D}_L V_{\mathrm{CKM}}^\dagger (m_u^{\mathrm{diag}})^2 V_{\mathrm{CKM}} m_d^{\mathrm{diag}} D_R$
  minimal number of spurions for a flavour transition.
- Leading term in $b \to s\gamma$: $\bar{s}_L V_{ts}^* V_{tb} m_t^2 m_b b_R$
- Right handed helicities suppressed by a power of the quark mass
- FCNC require at least two CKM matrix elements, at least one of which is off diagonal
- GIM: no FCNC's in case of degenerate quark masses

Flavour Symmetry: Leptons

- "Minimal field content" (no right handed neutrino)

$$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad E_R = e_R$$

- Smaller flavour group for the leptons

$$\tilde{G}_F = SU(3)_{E_L} \times SU(3)_{E_R}$$

- Transformations under $\tilde{G}_F$:
  $E_L \sim (3,1)$ and $E_R \sim (1,3)$
- Yukawa term for the leptons

$$L_{\text{Yuk}} = \bar{E}_L H Y_E E_R$$

- Y_E can be diagonalized by a $\tilde{G}_F$ transformation
  No flavour mixing for leptons !

Lepton Flavour Violation: Higher Dimensional Operators

- Dim-5 operator leading to a ν Majorana Mass Term

$$\mathcal{L}_{\rm Maj} = \frac{1}{2\Lambda_{\rm LN}} \left(N^T g N\right)$$

$$\text{with} \quad N = \left(T_3^{(R)} + \frac{1}{2}\right) H^\dagger L$$

$$\text{and} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & v + h_0 - i\chi_0 \end{pmatrix}$$

- $\Lambda_{\rm LN}$: Scale of lepton number violation
- g: New Spurion field transforming as $(\bar{6}, 1)$ under $\tilde{G}_F$
- Y_E, g can (in general) not be diagonal simultaneously

New Physics in MFV: Quarks

- Generic point of view: Consider the Standard model as an effective theory, valid at the electroweak scale
- “New Physics” enters below M_W through power suppressed operators with dimensions ≥ 6
- Assume that Y_U, Y_D (and Y_E) are still the only spurions explicitely breaking flavour
- The flavour transitions of the new-physics contributions are still suppressed by the same CKM factors and masses as in the Standard Model
- Focus first on quarks ...

Power Counting and Wolfenstein Parametrization

- Power Counting $\sim$ "small" symmetry breaking
- Implemented by the Wolfenstein parametrization

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.2$$

- Quark Masses (except top) are small compared to the electroweak scale
- Additional spurion insertions yields more suppression (except for $t \to b$ transitions, fllavour diagonal)
- Consider only minimal number of spurion insertions
  - Up to four insertions for right $\to$ right transitions

Effective Field Theory Picture of New Physics

- List the various quark transitions:

	U_L	U_R	D_L	D_R
$\bar{U}_L$	$V_{u_L}^\dagger Y_D Y_D^\dagger V_{u_L}$ $= V_{\rm CKM} \hat{m}_D^2 V_{\rm CKM}^\dagger$	$V_{u_L}^\dagger Y_D Y_D^\dagger Y_U V_{u_R}$ $= V_{\rm CKM} \hat{m}_D^2 V_{\rm CKM}^\dagger \hat{m}_U$	$V_{u_L}^\dagger V_{d_L}$ $= V_{\rm CKM}$	$V_{u_L}^\dagger Y_D V_{d_R}$ $= V_{\rm CKM} \hat{m}_D V_{\rm CKM}^\dagger$
$\bar{U}_R$	h.c.	$V_{u_R}^\dagger Y_U^\dagger Y_D Y_D^\dagger Y_U V_{u_R}$ $= \hat{m}_U V_{\rm CKM} \hat{m}_D^2 V_{\rm CKM}^\dagger \hat{m}_U$	$V_{u_R}^\dagger Y_U^\dagger V_{d_L}$ $= \hat{m}_U V_{\rm CKM}$	$V_{u_R}^\dagger Y_U^\dagger Y_D V_{d_R}$ $= \hat{m}_U V_{\rm CKM} \hat{m}_D V_{\rm CKM}^\dagger$
$\bar{D}_L$	h.c.	h.c.	$V_{d_L}^\dagger Y_U Y_U^\dagger V_{d_L}$ $= V_{\rm CKM}^\dagger \hat{m}_U^2 V_{\rm CKM}$	$V_{d_L}^\dagger Y_U Y_U^\dagger Y_D V_{d_R}$ $= V_{\rm CKM}^\dagger \hat{m}_U^2 V_{\rm CKM} \hat{m}_D$
$\bar{D}_R$	h.c.	h.c.	h.c.	$V_{d_R}^\dagger Y_D^\dagger Y_U Y_U^\dagger Y_D V_{d_R}$ $= \hat{m}_D V_{\rm CKM}^\dagger \hat{m}_U^2 V_{\rm CKM} \hat{m}_D$

- Loops may change the number of insertions:
  Suppressed by powers of Wolfenstein λ

New Physics in MFV: Leptons

- Majorana term is a “new physics” contribution
- Distinguish between the scale of lepton flavour violation and lepton number violation
- For dim-6 operators: Possible Spurion combinations

$$g^\dagger \times g \sim \bar{6} \times 6 = 1 + 8 + 27$$

- Bilinears (e.g. $\tau \to \mu\gamma$) are governed by $\Delta = (g^\dagger \times g)_8$ $\to$ predicts e.g. relations between $\tau \to \mu\gamma$ and $\tau \to e\gamma$
- Four fermion operators for e.g. $\tau \to \mu\mu\mu$ can have a contribution of the 27-plet
- Even in MFV no relation between $\tau \to e\gamma$ and $\tau \to e\mu\mu$

Flavor Models

Top-down instead of bottom-up

- How to get an Idea about the mass matrices:
  - Guess some matrices with as few parameters as possible:
    "Textures" as many zeros as possible
  - Use some symmetry to obtain (at least qualitatively) some insight into mass matrices
    e.g. a simple horizontal $U(1)$
- „, or are the parameters "just so"?

Textures: Two Family Example

- Find two matrices $\mathcal{M}_u$ and $\mathcal{M}_d$ with less than five parameters
- $\to$ Relation(s) between m_u, m_c, m_d, m_s and Θ_C
  Simplest guess: Diagonal $\mathcal{M}_u$ and nondiagonal $\mathcal{M}_d$

$$\mathcal{M}_u = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} \qquad \mathcal{M}_d = \begin{pmatrix} 0 & a \\ a & 2b \end{pmatrix}$$

- Matrix diagonalizing $\mathcal{M}_d$ is already the CKM Matrix
- Four Parameters $\to$ One relation !
- Model predicts $\tan \Theta_C = \sqrt{\frac{m_d}{m_s}}$ (which is not bad!)

- Has been done also for three families
- Guesses often supported by assuming (discrete) symemtries
- Typical structure: $\tan\theta_{ij} \sim \sqrt{m_i/m_j}$
- Remains Guesswork, until some deeper understanding of the guesses emerges.

Flavour Invariants

- Precise form of the mass matrices depend on the basis choice in Flavor space
- Basis independent statements only as a relation between Invariants
- For the two-family case these are e.g.

$$I_1 = \mathrm{Tr}(Y_U Y_U^\dagger) \quad I_2 = \mathrm{Tr}(Y_D Y_D^\dagger)$$
$$I_3 = \mathrm{Tr}(Y_U Y_U^\dagger Y_U Y_U^\dagger) \quad I_4 = \mathrm{Tr}(Y_D Y_D^\dagger Y_D Y_D^\dagger)$$
$$I_5 = \mathrm{Tr}(Y_D Y_D^\dagger Y_U Y_U^\dagger)$$

- There are as many independent Invariants as there are physical parameters

Conclusion on New Physics in Flavor

- Unlike in the gauge sector we do not have a guiding principle to construct a theory of flavor
- CKM as well as MFV is just a parametrization of ignorance
- No new physics model explainig flavor
- ... maybe wit the exception of some “Frogatt Nielsen like” models
- can the parameters be “just so”?

Overall Conclusions

- BaBar and Belle established the CKM picture of flavor
- LHCb is running, squeezing the SM further:
  - $B_s \to \mu\mu$
  - $B \to K^* \ell\ell$
  - V_{ub} measurements
- Belle 2 is upcoming:
  - Factor of 10 or 20 more data
  - Significant increase in precision
  - Look for rare and impossible decays
- At the end:
  Possible a clear indirect hint to BSM physics?
- But how can we finally tell, if the scales are really very high?